Energy-Efficient Power Control in Wireless Queueing Networks *

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Abstract

In this paper we consider the problem of power control of packet data transmission in a two user multiple access system, with joint consideration of average energy per packet and delay per packet. The problem we consider is an interacting queue problem, where the packets arrive randomly to the buffers of infinite capacity and the transmission success probability of one node depends on whether the other node is transmitting or not. Our aim is to understand as much as possible, the characteristics of the optimal power control policy. As a first step we consider a number of simple multiple access systems and make a comparison of their energy and delay performances. Then we consider dynamic control of power, and formulate the optimal power control problem.

1 Introduction

Power and rate control are two adaptive transmission techniques that are useful for adapting to the network conditions in order to share and use resources efficiently. Power control was first studied and applied in networks that carry *voice* traffic, with an aim of compensating for channel fluctuations (fading) and interference. For this reason SINR balancing power control algorithms were proposed [1]. More recently, power and rate control were applied in *data* communications in order to adapt the rate to fading and interference in order to maximize the throughput with constraints on power. Rate control is the control of transmitted bits per unit time, and it can be achieved by techniques like adaptive modulation and symbol time control. For example in papers [2] and [3] rate and power is jointly adapted in CDMA systems in order to maximize throughput subject to constraints on instantaneous power and rate.

The power/rate control schemes proposed in the above studies exploit the delay tolerance of data transmission and encourage the transmission by node(s) with the best channel conditions at each time in order to maximize the throughput. Unfortunately this can be prohibitive in terms of fairness and delay performance, because some nodes may never achieve the best channel condition. Most of the previous works on power/rate control disregard the random arrival of

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data packets and do not consider delay and buffer overflows (queueing cost) caused by the queueing of the packets.

The aim of our research is to develop power control schemes for wireless transmission scheduling that jointly address energy and *queueing cost* in a multiple access queueing system with random packet arrivals. In a previous paper [4], we considered a single node system, with a finite buffer and random packet arrivals. We jointly considered energy expenditure and buffer overflow as performance criteria and studied the control of power as a function of queue size. Joint consideration of energy and queueing cost in a single user setting suggests some novel trade-offs. However with the wide-spread use of wireless devices, it is a more interesting and challenging to consider multiple users sharing a single channel, which requires the coordination of those users, by jointly considering the energy expenditure of the nodes and queueing cost. This requires the connection of multiple access and queueing and the resulting system becomes a system of *interacting queues*. In these systems the success probability of transmissions depend on whether nodes transmit simultaneously or alone. These kind of systems were previously analyzed in the context of ALOHA-type systems. For two user collision channels, exact expressions for average delay were found for symmetric systems by Sidi and Segall [5]. More recently Naware and Tong extended these works for average delay of a symmetric system with capture properties in [6]. For systems of $N > 2$ users, Ephremides et. al. computed some bounds on average delay for the collision channel [7, 8]. Previous works on this subject assumed either collision or capture channels and only a few of them could find an exact expressions for average delay. Such channel models are not appropriate for CDMA systems. It is actually possible that both users to be successful in the event of simultaneous transmission, whereas failure is also possible even if a node is transmitting alone.

The rest of the paper is organized as follows. In Section 2 we summarize our previous work on a single user system in [4]. In Section 3, as a first step in solving the multiple access problem, we analyze the energy and delay performances of a two user multiple access systems, where the packets arrive at infinite capacity buffers according to a Bernoulli arrival process. Then we simulate and compare the performances of four different system models in terms of delay and energy. We then formulate the optimal power control problem for finite queues and compute the optimal policy.

2 Single User System

In [4] we considered a wireless link with random message arrivals and a finite capacity buffer, and addressed the problem of jointly minimizing energy expenditure and queue overflow by dynamically choosing from two alternative power levels. Figure 1 illustrates the system considered in the paper.

The general assumptions on the system is as follows. A source supplies a wireless node with packets of length L that arrive according to a Bernoulli process of rate $0 < \lambda < 1$ packets/time slot. Arriving packets enter in the queue that has a capacity of K packets, where the packets arriving in excess of K packets are lost. Packets are transmitted by using BPSK modulated signals of duration T_s/L, where T_s is also equal to the time slot length ($R_s = L/T_s$ is the symbol rate). P is the power that is used in transmission, and PT_s is the energy per single transmission. We do not consider forward error control (FEC), although some basic form of it can be incorporated in a straightforward way. Instead, a simple ARQ retransmission protocol is assumed to be used. There is an error-free feedback channel with zero feedback delay and unless all of the symbols of the transmitted packet are error-free, they are retransmitted. The number of retransmissions is therefore geometrically distributed. The received signal is sub-

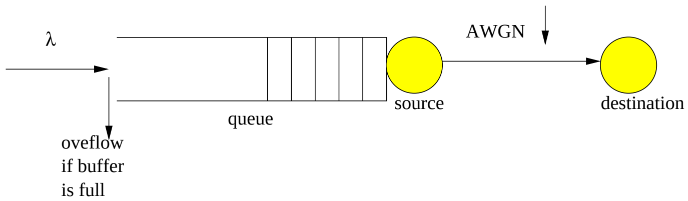

Figure 1: Single user queueing system model

ject to additive white gaussian noise of power spectral density N_0. Therefore, the probability of successful packet transmission is equal to $\mu = \left(1 - Q\left(\frac{PT_s}{N_0}\right)\right)^L$. In this system lower transmission power implies less energy per single packet transmission attempt. However, lower power yields greater average service time, and hence possibly more energy per successful packet transmission as well as higher probability of overflows.

In this setting, we considered the problem of dynamically choosing between two alternative power levels, P_1 and P_2 ($P_1 < P_2$) based on the size of the queue. Power decisions are made at every slot boundary. We modeled this system as a Markov Decision Process . Let $g(x_t, u_t)$ denote the one stage cost when the number of bits in the system is x_t and control u_t is applied. Now we will first define the single stage cost functions. It is a linear combination of energy and overflow costs. Energy cost ($E(x,u)$) is proportional to the energy expenditure in a time slot. Overflow cost $O(x,u)$ is proportional to the total bits that come in excess of the queue size during a time slot. They are given by,

$$E(x,u) = \begin{cases} P_u T_s, & u = 1,2 \\ 0, & u = 0 \end{cases}, O(x,u) = \begin{cases} 0, & x < K \\ \lambda, & x = K \end{cases} \tag{1}$$

The overall one stage cost is the weighted sum of energy expenditure and overflow costs. Our aim was to find the policy that minimizes the infinite horizon discounted cost function. We proved by using an induction argument that the optimal strategy of selecting transmission power is of threshold type. That is, the optimal policy switches from small to large power at a queue size level.

3 Multiple Access System

Our aim in this paper is to extend our approach to the wireless multiple access environment with queueing. We consider uplink multiple access scenarios where, (initially) two wireless nodes are trying to transmit to a center node. The transmitted packets are of equal length and time is also divided into equal length time slots. Packets arrive at each node from uncorrelated sources according to Bernoulli distribution. Each node has initially an infinite capacity buffer and the arriving packets stay there until they are successfully transmitted. A node may start packet transmission only at the beginning of the time slots. Nodes are equidistant to the center node and they use a fixed power level in each transmission. Therefore the success probabilities are symmetric. As opposed to the classical collision channel, one or both nodes can be successful even if they transmit simultaneously, moreover a node may not be successful even if it is transmitting alone. This makes the analysis of these systems even harder then previously analyzed collision and capture channels and incorporates the physical layer properties. We start by considering the following systems shown in Figure 2. In order to analyze the system we first write the expression for the probability generating function of the queue sizes in terms of

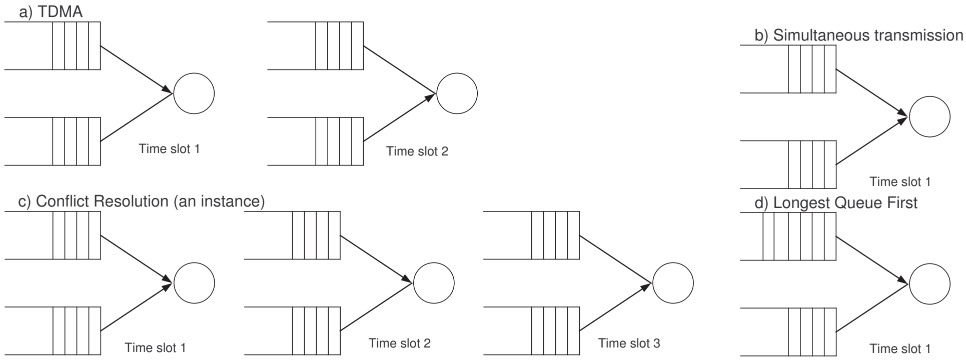

Figure 2: Multiple access systems: a) Time sharing b) Simultaneous Transmission c) Simple conflict resolution. d) Longest Queue First.

the boundary values and probability generating function of the packet arrivals. Then we solve a Riemann-Hilbert boundary value problem [5] in order to find the average queue size.

Let $A_i(t)$ be the number of packets arrived at node i in time slot t ($i = 1,2$). The processes $[A_i(t)]$ consists of Bernoulli random variables that are independent and identically distributed at each time slot t with rate r. Therefore we can define the generating function $A(x)$ as $A(x) = E[x^{A_i(t)}] = (rx+\overline{r}), i = 1,2$ where $\overline{r} = 1-r$.

3.1 Time Sharing System

The first system that we consider is a time sharing system, where the odd (even) time slots are reserved to node 1 (2) and each node transmits in its time slot, whenever it has an available packet. Therefore the system that we have can be simply decomposed into two single queue systems, where each attempted transmission is accomplished over two time slots, however energy is spent in only one of them. Probability generating function for the number of packets that arrive in two time slots is $F(x) = E[x^{A_i(t)+A_i(t+1)}] = (rx+\overline{r})^2, i = 1,2$.

Next we define the departure probabilities. Nodes simply share the time slots and they don't transmit simultaneously. Let p be the successful transmission probability. Let's also define $G(x)$ to be the steady-state joint generating function of the queue length.It is given by $G(x) = lim_{t\to\infty}E[x^{L_i(t)}], i = 1,2$, where $L_i(t)$ is the queue length of node i at time slot t respectively. Using the regular technique in [5] and [6] we can write the following equation in order to obtain the expression for $G(x)$:

$$G(x) = F(x)[G(0) + (G(x) - G(0))(x^{-1}p + \overline{p})] \quad (2)$$

From this equation we can write $G(x)$ in terms of $G(0)$. Then from $G(1) = 1$, we find that $G(0) = \frac{p-2r}{p}$. In order to maintain stability and obtain a steady state characteristics, the inequality $p > 2r$ must hold. Average queue size of node 1 can be derived by taking the derivative of $G(x)$ and evaluating at $x = 1$.

$$G'(1) = \frac{dG(x)}{dx}|_{x=1} = \frac{2r-3r^2}{p-2r} \quad (3)$$

Using the Little's result [9] we find the average delay per packet in time slots by dividing the above quantity by r. Let the transmission power be P units. Then the energy per successfully transmitted packet becomes $E = PT_s/p$, where T_s is the time slot length. Success probability p can be written as a function of transmission power.

3.2 Simultaneous Transmission

In the second system, both nodes can transmit whenever their queues are nonempty. For this system we can define the generating function of the joint packet arrival process, $F(x,y)$ as $F(x,y) = E[x^{A_1(t)} y^{A_2(t)}] = (rx+\bar{r})(ry+\bar{r})$, where $\bar{r} = 1-r$. We assume that the receiver structure at the center node allows the reception of multiple packets at the same time (e.g. a CDMA system). Let p be the probability of success when a node is transmitting alone and q be the probability of success when the other node is also transmitting. Obviously we assume $q < p$. The steady-state joint generating function of the queue lengths is $G(x,y) = lim_{t\to\infty} E[x^{L_1(t)} y^{L_2(t)}]$, where $L_1(t)$ and $L_2(t)$ are the queue lengths of node 1 and 2 at time slot t respectively. Using the regular technique in [5] and [6] we can write the following equation in order to obtain the expression for $G(x,y)$:

$$G(x,y) = F(x,y)\{G(0,0) + [G(x,0)-G(0,0)]A(x,y) + [G(0,y)-G(0,0)]B(x,y) \\ +[G(x,y)-G(x,0)-G(0,y)+G(0,0)]C(x,y)\} \quad (4)$$

where

$$A(x,y) = x^{-1}p+\bar{p}, \quad B(x,y) = y^{-1}p+\bar{p}, \quad C(x,y) = (x^{-1}q+\bar{q})(y^{-1}q+\bar{q}) \quad (5)$$

We will use the symmetry of the system in order to find the average queue size and delay. Using the facts $G(1,1) = 1$ and $G(0,1) = G(1,0)$, from (4), we can write the following.

$$q-r = G(1,0)(2q-p) + G(0,0))(p-q) \quad (6)$$

Now we will derive $G_1(1,1) = dG(x,y)/dx|_{x=y=1}$. To do this we first replace y by 1 and take the derive $dG(x,1)/dx$. We then take the limit of this expression as x goes to 1. Here we have to use L'Hospital's rule twice and then we use (6) in order to get the expression for $G_1(1,1)$.

Now we will also derive the expression for $G'(1,1) = dG(x,x)/dx|_{x=1}$. In order to do this, we replace y with x in equation (4), use L'Hospital's rule twice and also use Equation (6). Then using the fact that $G'(1,1) = G_1(1,1) + G_2(1,1) = 2G_1(1,1)$ we find another expression for $G_1(1,1)$. From these equations we eliminate $G_1(1,0)$ and find following expression for $G_1(1,1)$.

$$G_1(1,1) = \frac{-pr^2+2qr-qr^2+q^2(1+G(0,0)-2G(1,0)))(p-q)}{2p(q-r)} \quad (7)$$

Average delay expression can be obtained by dividing the above expression by r. Unfortunately we cannot completely get rid of the term $1+G(0,0)-2G(1,0)$. This quantity is in fact the probability that both queues are nonempty and it is naturally between 0 and 1. If the traffic load is higher, we can approximate it by 1 (hence upper bound), and when it is low, we can approximate it by 0 (lower bound). In Figure 3, we plotted the average queue size as a result of the simulation along with the derived upper and lower bounds versus arrival rate r. We see that the bounds are quite accurate and simulation results converge to lower bound for low rate and upper bound for high rate. In fact these are the first bounds derived for such systems.

Finding the energy per packet is also complicated because some of the transmissions of a packet are simultaneous and some are made alone, which affects the service time and complicates the energy per packet computation. If the traffic is very heavy, then in most of the slots two nodes will transmit simultaneously. In this case the success probability for each node is q and the average energy expenditure per successfully transmitted packet approaches PT_s/q. On the other hand if the traffic is light, then most of the transmissions will be made alone. In

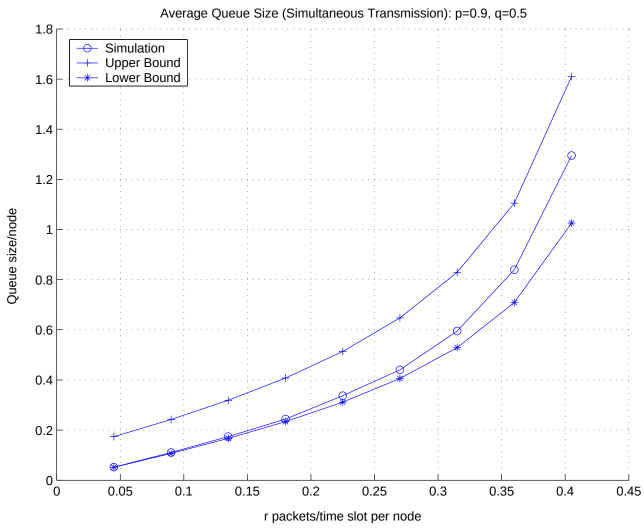

Figure 3: Comparison of the upper and lower bounds with the simulation results for the average queue size for the simultaneous transmission system.

this case the success probability is p and the energy per packet approaches PT_s/p. These two values constitute upper and lower bounds on energy expenditure however they are not good bounds.

3.3 Simple Conflict Resolution

The third multiple access system is inspired by the conflict resolution process in the classical collision channel [9]. Since we only consider two nodes, conflict resolution becomes very simple. In the first slot of the conflict resolution period (CRP) nodes transmit if they have packets. If no error occurs (i.e nodes are either idle or successful) then in the next slot a new CRP starts. If only one of the nodes transmit with error (i.e. the other is either idle or successful), then the erroneous node retransmits in the next slot; in the third slot a new CRP starts. If both nodes transmit and fail, then the nodes share the next two time slots in a TDMA fashion as in the first system considered. A new CRP starts in the fourth slot.

Here we can construct an embedded Markov chain $[L_1'(t'), L_2'(t')]$, where $L_1'(t'), L_2'(t')$ are the queue size of nodes 1 and 2 at the slot boundaries corresponding to the beginning of a CRP. Process at $t+1'$ depends on the state at t'. Let $G(x,y)$ be the joint generating function of the queue sizes at the Markov epochs. G(x,y) is written as in Equation (4), where

$$A(x,y) = x^{-1}p + \overline{p}(x^{-1}p + \overline{p})F(x,y), \quad B(x,y) = y^{-1}p + \overline{p}(y^{-1}p + \overline{p})F(x,y) \tag{8}$$

$$C(x,y) = (x^{-1}q + \overline{q}(x^{-1}p + \overline{p})F(x,y))(y^{-1}q + \overline{q}(y^{-1}p + \overline{p})F(x,y)) \tag{9}$$

The derivation of average delay and detailed analysis for this system is quite complex and is not pursued further here. Note that a major complication is the fact that the end of the elementary conflict resolution period does not guarantee that the conflict has been resolved.

3.4 Longest Queue First

In the last multiple access system that we consider, the node with the longer queue is served at every time slot. This is also not an easy system to analyze; there are some analysis for the case of heavy traffic. However the energy per packet is easy to derive. Since every transmission is made alone, the success probability for each transmission is p. Therefore the energy per packet is PT_s/p.

4 Performance Comparison

In this section we will compare the energy and delay performances of the above systems. For this purpose we should model the physical layer in more detail in order to find the power levels corresponding to the success probabilities. We adopt the *uncoded* CDMA model in [3] and [10]. In this model the signature sequences are created randomly. We assume independent detection errors for the bits in a packet and use a Gaussian approximation for the interference. In fact Gaussian interference expression is valid for systems of large number of users, which is not the case for our two user system. However, this expression is initially chosen in order to simply relate the transmission powers to success probabilities. More realistic formulations will be used in the future work. The SINR for each bit corresponding to user 1 is as follows:

$$SINR_1(P_1,P_2) = \frac{W}{R}\frac{P_1}{P_2+N_0W} \tag{10}$$

Here W is the bandwidth (chips per second), R is the bit rate and W/R is the chips per bit. If node 1 is transmitting alone, then $P_2 = 0$. At each time slot, a packet of L bits is transmitted and the transmission is considered as successful if every bit is received correctly [3]. BPSK modulation is used for each symbol, and therefore the probability of bit error is:

$$E_b = Q(\sqrt{SINR_1}) \tag{11}$$

Packet success probability is therefore given by;

$$S_p = (1-Q(\sqrt{SIR_1}))^L \tag{12}$$

4.1 Simulations

We assume packets of $L = 100$ bits. Noise power $\sigma^2 = N_0W$ is taken to be equal to 1 unit and spreading gain is taken to be 4. We run each simulation for 10000 packets and take the average of 10 simulations for each point in the graph. Figures 4 (a) and (b) show the delay and energy characteristics of these four systems versus packet arrival rate. Each user transmits with power 1.0 unit. We see that longest queue first scheme (LQF) is the best in terms of delay and energy. We also see that conflict resolution scheme performs comparably well with respect to longest queue first. Time sharing performs worst in terms of delay because it doesn't utilize the time slots efficiently. The schemes differ in terms of the feedback information that is required. In order to apply LQF both nodes should report their queue sizes to the center node, whereas only the success and failure of the nodes is sent as feedback order to apply the Conflict Resolution.

Figures 5 (a)and (b) show the delay and energy characteristics of these four systems versus packet arrival rate for 0.8 unit of transmission power. Since the transmission power is smaller, the success probabilities get smaller and delay increases. This especially affects the Simultaneous Transmission scheme, because the simultaneous success probability reduces abruptly as the transmission power decreases. Conflict Resolution scheme performs still comparably well with respect to the Longest Queue First scheme. As for the energy performance, Simultaneous Transmission scheme performs worse since too much power is wasted because of the low success probabilities. On the other hand in other schemes energy per packet is decreased when the

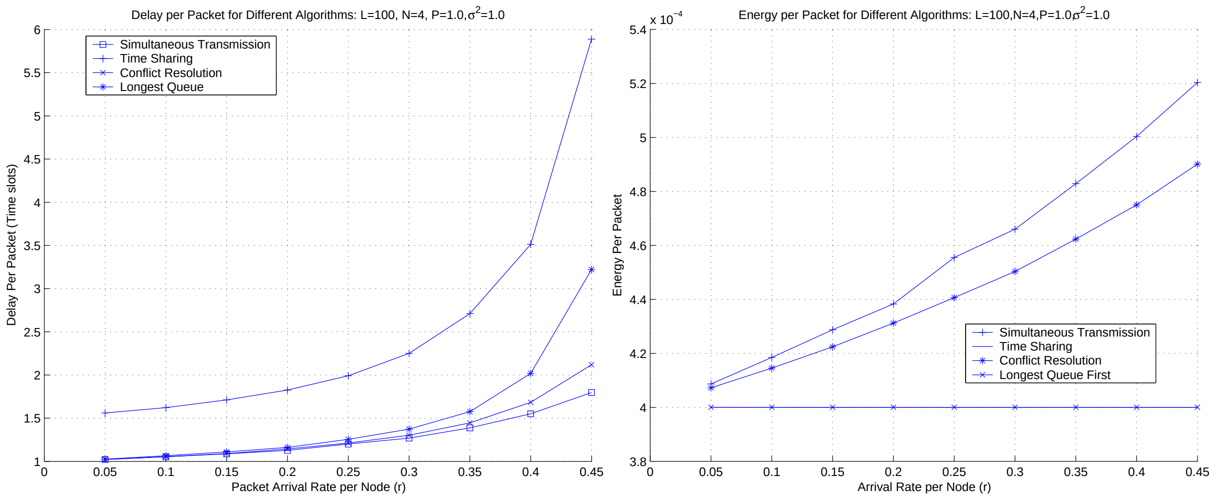

Figure 4: Comparison of performances for $P = 1.0$: a) delay b) energy

power is decreased. As it is observed in our previous paper [4], decreasing power can be used as a way to decrease energy per packet, but this should be done together with a more intelligent multiple access scheme.

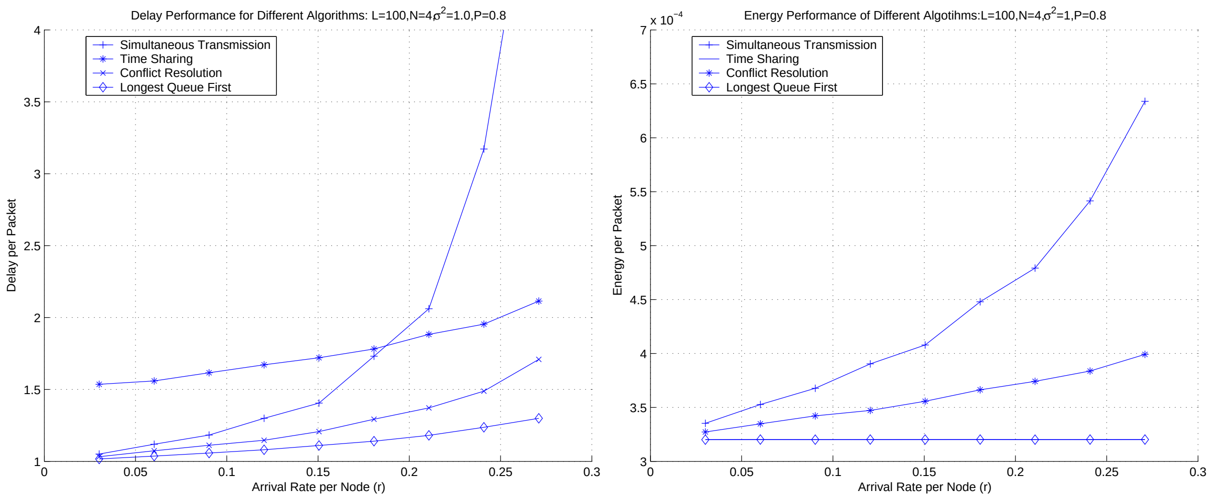

Figure 5: Comparison of performances for $P = 0.8$: a) delay b) energy

5 Formulation of the Optimal Control

Lastly we will formulate the problem of power control in a two user multiple access setting. In this part consider nodes with finite buffer capacity of K packets. The state of the system at time slot t is the queue sizes of the two nodes, $\overline{x^t} = (x_1^t, x_2^t) \in \{0, 1, \ldots, K\} \times \{0, 1, \ldots, K\}$. The control action in the system, $\overline{u^t} = (u_1^t, u_2^t)$ is the power levels of the two nodes, where at each time slot they are chosen from a set of power levels $\{0, P_1, P_2\}$. When queue size is zero only the power 0 can be used. Transition probabilities from one queue state to another depends on the transmission powers and packet arrival rate. They are denoted as $p_{x_1,x_1'}(u_1,u_2)$ and $p_{x_2,x_2'}(u_1,u_2)$ and they can easily be calculated. The cost incurred at a single stage (time slot) is a weighted sum of the energy spent in that time slot and the total queue size of the system and buffer overflows. It is given as,

$$g(\overline{x},\overline{u}) = c_1(u_1+u_2)+c_2(x_1+x_2)+c_3\lambda(I\{x_1=K\}+I\{x_2=K\}) \quad (13)$$

where $I\{.\}$ is the indicator function. We define a set (Γ) of policies, such that when transmitting the head of the line packet for the first time at time $t+1$, a policy $\gamma \in \Gamma$ uses the information $\overline{x^t}$, in order to select the transmission power. A policy γ^* is optimal among the policies in Γ if it minimizes the discounted cost function,

$$V_\alpha = E_{\overline{x}}^{\gamma} \sum_{t=0}^{\infty} \alpha^t g(\overline{x^t},\overline{u^t}) \quad (14)$$

where α is the discount factor and $E_{\overline{x}}^{\gamma}$ is the expectation with respect to x when policy γ is used. This problem can be solved by solving the following Bellman equation by value iteration.

$$V(\overline{x}) = \min_{\overline{u}}\{g(\overline{x},\overline{u})+\alpha\sum_{x_1'}\sum_{x_2'} p_{x_1,x_1'}(u_1,u_2)p_{x_2,x_2'}(u_1,u_2)V(\overline{x'})\} \quad (15)$$

We computed the optimal policy by solving the Dynamic Programming Equations for a number of parameters. Figures in 6 show the optimal policy for node 1 and 2 for the parameters $P_1 = 0.8$,$P_1 = 1.0$, $c_1 = 1$,$c_2 = 0$, $c_3 = 30$, where (a) is for $\lambda = 0.4$ and (b) is for $\lambda = 0.6$. In (a) the nodes transmit only when they are very close to overflow in order to save energy. In (b) the nodes transmit at lower queue sizes since the arrival rate is higher.

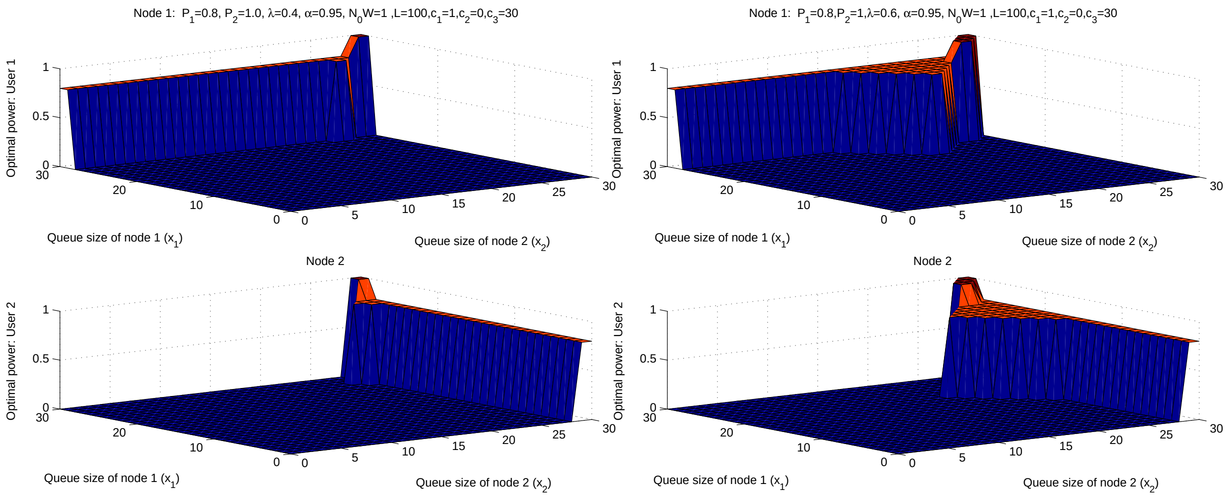

Figure 6: Optimal Power Control for $P_1 = 0.8$,$P_1 = 1.0$, $c_1 = 1$,$c_2 = 0$, $c_3 = 30$, a) $\lambda = 0.4$ b) $\lambda = 0.6$

Figures in 6 show the optimal policy for node 1 and 2 for the parameters $P_1 = 0.8$,$P_1 = 1.0$, $\lambda = 0.4$, where (a) is for $c_1 = 2$,$c_2 = 0$, $c_3 = 10$ and (b) is for $c_1 = 1$,$c_2 = 0.1$, $c_3 = 30$. In (a) the nodes don't use the large power level since the weight of the energy cost is increased. In (b) the nodes transmit at lower queue sizes since the cost for queue size is also included in the total cost. We see that the optimal multiple access is a combination of longest queue first and simultaneous transmission schemes where the transmission powers are also controlled according to the queue size.[1]

[1]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

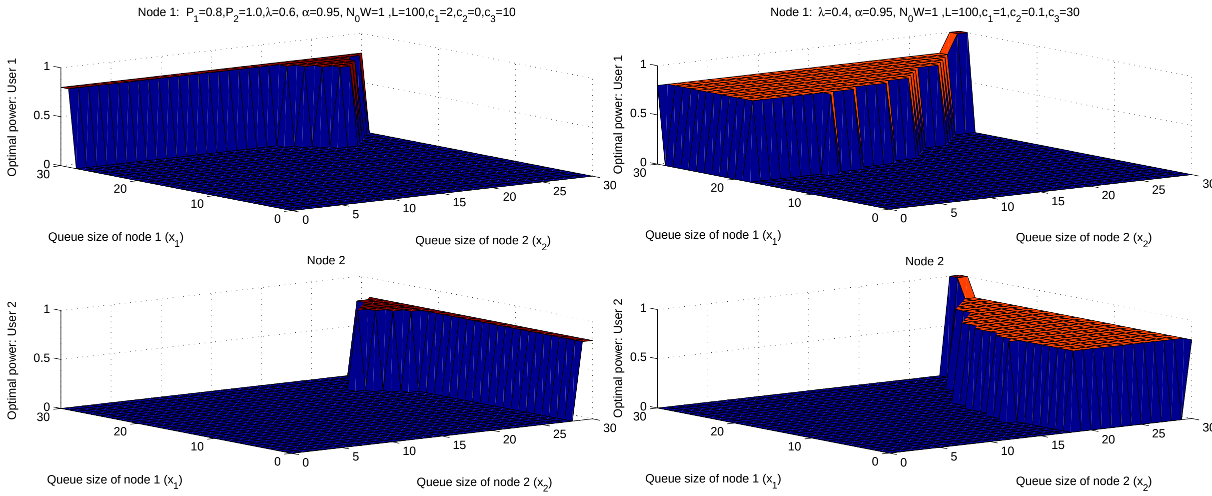

Figure 7: Optimal Power Control for $P_1 = 0.8$, $P_1 = 1.0$, $\lambda = 0.6$, a) $c_1 = 2$, $c_2 = 0$,$c_3 = 10$ b) $c_1 = 2$,$c_2 = 0.1$, $c_3 = 10$

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