# Minimum-Outage Broadcast in Wireless Networks with Fading Channels

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***Abstract*—We consider the problem of cooperative broadcasting for minimum outage in wireless networks. We consider wireless multihop broadcast as a set of transmitters that transmit in a certain order. The receiving nodes are able to combine all the previous transmissions and achieve better received SNR. We consider the problem of finding the optimal set and order of (fixed-power) transmitters that minimizes the overall outage probability in a wireless fading channel. We found the optimal solution using a Branch-and-Bound approach and proposed some suboptimal algorithms. The numerical results show that a simple algorithm that we propose performs surprisingly well.**

***Index Terms*—Wireless networks, broadcast, multicast, outage.**

## I. Introduction

THE objective of wireless broadcasting is to broadcast data reliably to all nodes in a network. Most of the previous works on wireless broadcast focused on energy efficiency. A wireless node can always increase its power to cover more nodes, which may be more efficient than reaching those nodes by multihop transmissions [1]. However, with multihop transmissions, a node has chance to overhear and combine multiple copies of the same packet, exploiting *cooperative diversity*. Maric and Yates [2] proposed an energy-efficient cooperative route selection method, where a node collects energy during each retransmission before it retransmits. After collected energy exceeds a required threshold, it reliably gets the transmitted data. However, nonfading and fixed channel state is assumed, which is unrealistic. In [3], [4] minimum-power broadcast and unicast routing were studied, however the proposed methods involve many-to-one transmissions , which require advanced synchronization, and are hard to implement.

In a fast fading environment, it is hard to know the exact channel condition all the time. Even if we measure the channel, the measurement can be invalid by the time transmission starts. A wireless link is subject to multipath fading. Signal to noise ratio (SNR) falling below a required threshold (i.e. *outage*) is a dominant cause of error in wireless communications. In [5], algorithms were developed for selecting the optimal unicast route based on outage probability as the reliability metric. Also in [6] power optimization is made under outage constraints to find an optimal cooperative unicast route. However , these two works consider unicast routing rather than broadcast or multicast. The authors in [7] propose a two stage cooperative multicasting protocol. However, there is a total power constraint instead of individual power constraints, and pathloss is not considered, which are unrealistic assumptions.

End-to-end (overall) outage probability (the probability of at least one of the destination nodes can not reliably decode the message) is a good metric to characterize reliability of multicast and broadcast algorithms. In this work, we consider a wireless network where a subset of nodes transmit in a certain order and with a fixed power. We consider the problem of finding the optimal transmission order of nodes, in order to minimize the end-to-end outage probability. As in [2], a node in the network hears and combines all signals from all of the transmissions done before. To find the optimal transmission order, we implemented a Branch-and-Bound [9] type of algorithm. We also propose greedy suboptimal algorithms and see if these algorithms show comparable performance.

## II. System Model

The source is located at the center of a circle and a number of receivers are located randomly inside the circle. The channel model includes pathloss and Rayleigh fading. The pathloss values are fixed and they are assumed to be known by the source node. Let $g_{i,j}$ and $h_{i,j}$ be the pathloss and Rayleigh fading between nodes i and j. The order of transmissions of nodes are determined centrally. [1] In order to concentrate on broadcasting problem we disregard the issues of interference, collision and multiple access. [2] Each node transmits at most once, and the total number of transmissions is limited to $T_{max}$. We aim to find, $\mathcal{O}$, which is the ordered set of transmissions and let $\mathcal{O}_i \subset \mathcal{O}$ be the ordered set of nodes that transmit before node $i$. Please note that if $i \notin \mathcal{O}$ then $\mathcal{O}_i = \mathcal{O}$ (every transmitter transmits before $i$). Let $\Gamma_i$ be the accumulated SNR at node i, which can be written as

$$\Gamma_i = \frac{P}{N_o W} \times \sum_{n=1}^{|O_i|} g_{n,i} h_{n,i} \tag{1}$$

where $P$, $N_o$ and $W$ are the transmission power, noise p.s.d. and bandwidth, which are same and fixed for all users.

There is a target rate $R_0$ and the signal to noise ratio required to achieve that rate is found using the Shannon capacity function $R_0 = W \log_2(1+\Gamma)$ bps. Let's define $e_0 = (2^{\frac{R_0}{W}} - 1)\frac{N_o W}{P}$. The successful reception probability of node $i$ (i.e. $\Pr(\sum_{n=1}^{|O_i|} g_{n,i} h_{n,i} > e_0)$) for a given transmission order can be written as [8]

$$P_s^i(\mathcal{O}_i) = \left( \prod_{n \in \mathcal{O}_i} \frac{1}{g_{n,i}} \right) \sum_{j \in \mathcal{O}_i} \frac{g_{j,i} e^{(-\frac{e_0}{g_{j,i}})}}{\prod_{k \in \mathcal{O}_i, k \neq j} (\frac{1}{g_{k,i}} - \frac{1}{g_{j,i}})} \tag{2}$$

[1]Finding the distributed versions of our algorithms is a future direction.

[2]Normally, because of interference constraints, number of simultaneous transmissions are limited and time should be shared among different scheduling instants. A scheduling or multiple access solution can be jointly considered. This is beyond the scope of this work, but is a future research direction.

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Overall outage probability can be written as,

$$P_o(\mathcal{O}) = 1 - \prod_{i=1}^{N} P_s^i(\mathcal{O}_i) \quad (3)$$

## III. Optimum Transmission Order

The problem here is to determine the order of $T_{max} - 1$ transmissions (after the source node) so that the overall outage probability is minimized. There are a total of $\frac{(N-1)!}{(N-T_{max})!}$ possible orders. Comparing all the possible orders in order to find the outage-minimizing one is prohibitively time consuming. Instead, we propose a solution based on the Branch-and-Bound technique [9]. This technique is especially useful for combinatorial problems with integer variables such as our problem. The algorithm involves a branching procedure, where a branch is an ordered set of transmissions, which can be *split* by adding another transmission to the existing order [3]. Let $\mathcal{B}$ be the set of branches. Maximum depth of the tree is $T_{max}$. Each branch is associated with a lower and upper bound, for the outage probability. At each step of the algorithm the branch with minimum lower bound is found and further splitted. If the upper bound for a branch is less than the lower bound of another branch, then the latter is *pruned*, which narrows down the search space. For a given ordered set of transmissions $\mathcal{O}$, the lower and upper bounds on outage probability are,

$$\texttt{LB}_{\mathcal{O}} = \begin{cases} 1 - \prod_{i=1}^{N} P_s^i(\mathcal{O}_i) & \texttt{if } |\mathcal{O}| = T_{max} \\ 1 - \prod_{i\in\mathcal{O}} P_s^i(\mathcal{O}_i) & \texttt{else} \end{cases} \quad (4)$$

$$\texttt{UB}_{\mathcal{O}} = 1 - \prod_{i=1}^{N} P_s^i(\mathcal{O}_i) \quad (5)$$

$\forall\mathcal{O} \in \mathcal{B}$, where $|\mathcal{O}|$ is the cardinality of the set $\mathcal{O}$. For $|\mathcal{O}| < T_{max}$ the lower bound is the outage probability, as if the nodes in set $\mathcal{O}$ are the only ones in the network. Upper bound corresponds to the case that the ordered set of transmissions in $\mathcal{O}$ are the only ones and no other transmissions will be done. As we go from the root to any leaf of the tree, the lower bound increases and the upper bound decreases. For $|O| = T_{max}$ the upper and lower bounds are equal.

The pseudocode is shown in Algorithm 1. When the branch with the lowest lower bound has equal upper and lower bound, then any other branch can be pruned, and the sole remaining branch is optimal (Line 6). Otherwise the best branch is further splitted (Lines 8-17). Pruning is done in Lines 13-15.

## IV. Suboptimal Algorithms

The first suboptimal algorithm Greedy 1 works in steps. At each step the algorithm tries adding each non-transmitting node to the existing order and calculates the outage probability (Lines 3-6). Then finds the node that, corresponding to the smallest overall outage probability (See Line 7). Speaking in terms of the BBB terminology, Greedy1 algorithm finds the node that results in the least upper bound (5).

In all algorithms Equation (2) is used in order to compute the node success probability. The runtime of the algorithms are roughly proportional to the number of times that (2) is calculated. In Greedy1, at the $i^{th}$ step there are N-i possible nodes to add to the order and for each of these the resulting overall outage probability is calculated using (3). For the nodes that are already transmitting, there is no need to calculate the node success probability again and again. So, equation (2) is called $X = \sum_{k=N-T_{max}+1}^{N-1} k^2$ times.

[3]The root of the tree is $\{1\}$ and it can be splitted to branches $\{1, 2\}$, $\{1, 3\}$, ..., $\{1, N\}$. Branch $\{1, 2\}$ can be further splitted into $\{1, 2, 3\}$, $\{1, 2, 4\}$, ..., $\{1, 2, N\}$

**Algorithm 1** Broadcast Using Branch and Bound (BBB)

1: Set $\mathcal{B} = \{\{1\}\}$ and calculate $\texttt{LB}_{\{1\}}$ and $\texttt{UB}_{\{1\}}$
2: **while** true **do**
3: find $b^* = \arg\min_{b\in\mathcal{B}}\{\texttt{LB}_b\}$
4: **if** $\texttt{LB}_{b^*} = \texttt{UB}_{b^*}$ **then**
5: $\texttt{bestbranch} = b^*$ and $\texttt{minoutage} = P_o(b^*)$
6: return
7: **else**
8: **for** $\forall\ n \notin b^*$ **do**
9: Form new branch by adding $n$: $b = \{b^*, n\}$
10: Calculate $\texttt{LB}_b$ and $\texttt{UB}_b$ using (4), (5)
11: **if** $\nexists b' \in B$ s.t. $\texttt{LB}_b > \texttt{UB}_{b'}$ **then**
12: Add the branch to the set $\mathcal{B} = \mathcal{B} \cup b$
13: **if** $\exists b' \in \mathcal{B}$ s.t. $\texttt{UB}_b < \texttt{LB}_{b'}$ **then**
14: Prune branch $b'$: $\mathcal{B} = \mathcal{B}/b'$
15: **end if**
16: **end if**
17: **end for**
18: **end if**
19: **end while**

**Algorithm 2** Greedy1

1: Set $O = \{1\}$
2: **for** $i = 2$ to $T_{max}$ **do**
3: **for** $\forall n \notin \mathcal{O}$ **do**
4: Set $\mathcal{O}'(n) = \{\mathcal{O}, n\}$
5: Calculate $P_o(O'(n))$ according to (3)
6: **end for**
7: Find $n^* = \arg\min_{n\in\mathcal{O}^c}\{P_o(\mathcal{O}'(n))\}$
8: $\mathcal{O} = \{\mathcal{O}, n^*\}$
9: **end for**

**Algorithm 3** Greedy2

1: Set $O = \{1\}$
2: **for** $t = 2$ to $T_{max} - 1$ **do**
3: Calculate $P_s^n(\mathcal{O}'(n)), \forall n \notin \mathcal{O}$ using (2)
4: Find $n^* = \arg\min_{n\notin\mathcal{O}'}\{P_s^n(\mathcal{O}'(n))\}$
5: $\mathcal{O} = \{\mathcal{O}, n^*\}$
6: **end for**
7: Set $\mathcal{O}'(n) = \{\mathcal{O}, n\}$ and calculate $P_o(O'(n)), \forall n \notin \mathcal{O}$ according to (3)
8: Find $n^* = \arg\min_{n\notin\mathcal{O}}\{P_o(\mathcal{O}'(n))\}$
9: $\mathcal{O} = \{\mathcal{O}, n^*\}$

In Greedy2, a simpler algorithm, the order is initialized with the source node. Then at each step, the success probabilities of all other nodes are calculated and the one with the highest success probability is added to the order (Line 2-6). This loop is continued until $T_{max} - 1$ nodes are added to the order. For

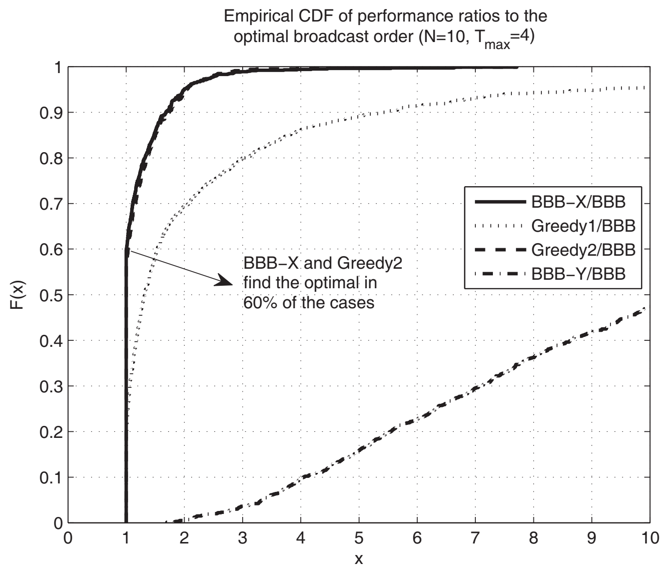


Fig. 1. CDF of the outage ratios to the optimal policy($N = 10, T_{max} = 4$).

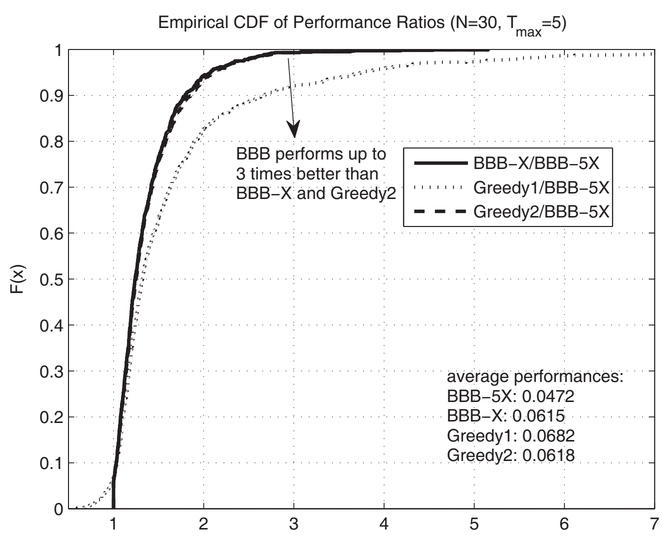


Fig. 2. CDF of the outage ratios to the BBB-5X(near optimal) policy($N = 30, T_{max} = 5$).

the last transmitter, the overall outage probabilities by adding each of the remaining $N - T_{max} + 1$ nodes are calculated and the node corresponding to the lowest overall outage probability is added to the order. In this algorithm (2) is called $Y = (N - T_{max} + 1)(N - T_{max}) + \sum_{k=1}^{T_{max}-1} N - k$ times for $T_{max} > 1$. In terms of the BBB terminology, Greedy2 algorithm at each step finds the node that minimizes the lower bound (4).

## V. Numerical Evaluations

In Fig. 1 we considered a 10-node system. The number of transmitters is $T_{max} = 4$. The considered algorithms are BBB, Greedy1 and Greedy2. For comparison, we also considered BBB-X and BBB-Y, where the regular Branch-and-Bound procedure is stopped when the number of node success (2) computations exceed X and Y defined above, respectively ($X > Y$). For each policy, we generated 1000 random networks and calculated the overall outage probabilities with each of the algorithms. For each algorithm we obtained an outage vector of length 1000. We divided (entry-by-entry) the vector of each algorithm to that of BBB (optimal) and plotted the empirical CDF. This gives us the *CDF of the performance degradation* of each algorithm with respect to the optimal one. The solid line in Fig. 1 show that BBB can perform up to three times better than BBB-X . Surprisingly, Greedy2 (although much simpler than Greedy1) performs almost identical to BBB-X (dashed line). It finds the optimal order in $60\%$ of the cases. BBB-Y on the other hand performs very poorly (dotted-dashed line), since Y is small and insufficient for Branch and Bound in finding a good transmitter order.

In Fig. 2 we consider a larger network of 30 nodes and $T_{max} = 5$. Because of the computational time, instead of BBB, we consider BBB-5X, where (2) is computed $5X$ times. Again we see that BBB-5X has up to three times better performance than BBB-X algorithm. Greedy2 performs almost identical to BBB-X and better than Greedy1. We also observe that Greedy1 performs better than BBB-5X with $10\%$ probability. However it can also perform up to 7 times worse than it and on the average it has the worst performance.

## VI. Conclusions

The numerical results show that the simple greedy suboptimal algorithm that we proposed performs surprisingly well with much faster running time. The proposed optimal and suboptimal algorithms can also be used in multicasting, as well as broadcasting. Future work will consider distributed implementations, power control and multiple access issues.

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