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Joint Power, Subcarrier and Subframe Allocation in Multihop Relay Networks

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SUMMARY

In this paper we study the problem of subframe, subchannel and power allocation in OFDMA-based multihop relay networks. The system consists of a base station (BS), a number of relay stations (RS) and mobile stations (MS). We consider frame by frame scheduling, where the frame is divided into two subframes such as BS-RS and RS-MS subframes. We study two different problems, satisfying link rate requirements with minimum weighted total power and maximizing proportional fairness. For the first problem we find the optimal solution and also propose a less complex subframe and bandwidth allocation scheme with good performance. For the second problem, we propose an algorithm that outperforms an existing scheme with less feedback.

KEY WORDS: 802.16j, MMR, OFDM, relay, resource allocation, minimum power, rate constraint

1. INTRODUCTION

Broadband wireless access networks [1] are designed to provide cellular systems that support fixed and mobile users with heterogeneous and high rate traffic requirements. In such networks a single base station (BS) is projected to cover a cellular area of radius on the order of miles. In such a large area users at the cell edge often experience bad channel conditions. Moreover in urban regions shadowing by big buildings degrade the signal quality in some areas. Increasing the number of base stations is an expensive solution. Increasing the base station power doesn't improve the conditions since it also increases the intercell interference. Deploying relay stations (RSs) is a feasible solution since typical RSs are cheaper than BSs and they don't need their own wired backhaul; therefore easier to deploy. Standardization of relay-based broadband wireless access networks is carried out by the 802.16j Relay working group [2], [3].

In 802.16j-based networks OFDMA is the multiple access and transmission technique. In OFDMA basic resources are subchannels and power. Subchannels experience frequency selective fading, which

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takes different values for different users and subchannels. Therefore optimal allocation of these resources is crucial in reaching various objectives such as improving throughput, reducing power consumption or maximizing fairness. Because of the cost requirements, relay stations are envisaged to have only one interface, therefore they aren't able to transmit and receive simultaneously. Therefore access (BS-RS, BS-MS) and relay (RS-MS) transmissions are scheduled in a TDMA manner, where these two groups of transmissions occur in different subframes. This adds the dimension of subframe duration to the existing OFDMA subchannel and power allocation problem. In this work we study the problem of joint subframe, subchannel and power allocation in OFDM-based cellular networks with relay stations. We study two different problems. We first study the problem of minimizing total weighted transmission power of users subject to user rate constraints. Solving such a problem is especially useful for rate constrained transmissions such as real-time voice and video streaming sessions. The objective of power minimization can be useful to limit the intercell interference in downlink and it has an additional use of energy efficiency for mobile users in uplink. Secondly, we study the problem subframe, subchannel and power allocation for proportional fairness. Proportional fairness is a suitable balance between max-min fairness and maximum total throughput and it is implemented in Qualcomm's HDR systems [4].

This paper is organized as follows. We present the related work and highlight our contributions in Section 2. In Section 3, the system and frame model is presented. In Section 4 we formulate and solve the jointly optimal subframe and channel allocation problem. Then we present a simpler subframe and bandwidth allocation scheme in Section 5 and compare it with an existing scheme. In Section 6 we studied subframe and bandwidth allocation for fairness and perform numerical evaluation and comparisons. The paper is concluded in Section 7.

2. RELATED WORK AND CONTRIBUTIONS

Multiuser OFDM has been studied in detail for traditional cellular downlink and uplink scenarios before. The authors in [5] formulate the capacity-maximizing subchannel and power allocation problem and propose a suboptimal allocation algorithm that shows significant performance improvement with respect to static FDMA resource allocation. In [6] the authors optimally solve the capacity maximization problem and show that allocating each carrier to the user with the best channel on that carrier and then distributing the power to the carriers by waterfilling maximizes the capacity. Optimal allocation in the case of uplink is more complex because in this case there are individual power constraints instead of total. The work in [7] characterizes the capacity-maximizing downlink resource allocation and proposes a suboptimal allocation algorithm with close-to-optimal performance. The solutions for general objectives such as proportional fairness or quality of service (QoS) were studied, for example, in [8], [10] . In [8] optimal subchannel and power allocation is studied for a specific type of rate-based objective function. In [9] capacity maximization was studied subject to rate proportionality constraints. In [10] and references therein, queue length based subchannel allocation schemes were proposed. Power minimization subject to rate constraints is another problem that is studied in the context of OFDMA. For example in [11], for a fixed set of discrete modulation levels minimum-power allocation is studied. Optimal solution is found by integer programming. Because of the complexity, the problem is divided into two parts, which are modulation level decision for each user and subchannel allocation for fixed modulation level. The latter can be solved using linear programming, which still has high complexity. Some suboptimal algorithms were used in [11] in order to solve this problem. In [12] minimum-power resource allocation is studied for a continuous modulation set in multiuser

OFDM systems. Subchannel allocation is inherently a non-convex problem, however it is observed in [12] that the solutions using Lagrange dual relaxation method results in almost optimal performance especially as the number of subchannels increases. Subgradient methods such as Ellipsoid method [20] can be used to find the jointly optimal subchannel and power allocation. However, this method has slow convergence, which makes it necessary to find a good suboptimal algorithm.

On the other hand, resource allocation for cellular multiuser OFDM systems with relay stations have not been studied sufficiently. Previous work in the literature is mostly on throughput maximization. For example the work in [13] develops a convex optimization approach in order to maximize throughput subject to BS, RS or total power constraints. The authors in [14] and [15] propose heuristics that improve throughput and coverage. The paper [16] analyzes and simulates the uplink capacity of a relay assisted OFDMA system. Lastly, we have recently proposed a resource allocation scheme that jointly satisfies fairness and rate constraints by allocating subframes, bandwidth and power in [17], however in that work we assumed flat fading, which suggests simpler resource allocation schemes, however doesn't take advantage of frequency selective fading. The authors in [18] propose a proportional fair resource allocation scheme. Optimization of subframe durations for BS and RS subframes was not considered in [18]. In this work we proposed a scheme that determines subframe durations and that allocates bandwidth to individual relay stations in a fair manner. Centralized resource allocation schemes for cellular relaying systems require feedback from the RSs, which becomes complex when channel conditions for each user and subchannel has to be fed back. The proposed scheme is also suitable for semi-distributed operation and requires less communication between BS and RSs.

Minimum power resource allocation subject to rate constraints in relay systems was only studied in [19]. In this work the authors propose a subframe, subchannel allocation and bit loading scheme. They adopt the system model in [11] and divide the frame into BS and RS subframes. The authors propose a heuristic to determine the durations of these two subframes.Then the resource allocation problem becomes separate instants of minimum-power downlink allocation problems studied in [11]. Optimally dividing the frame is important because a linear change in transmission time results in an exponential change in power expenditure for a target rate. In this work we consider a continuous set of modulation and coding and a continuous SNR-rate relation unlike [19]. We first propose solution that jointly optimizes subframe time, subchannel and power allocations. We depart from the system model of [12] and extend it for a system with relays. The objective is to minimize total weighted transmission power of all links subject to link rate constraints. Considering the complexity of this solution, we divide the problem into two such as subframe determinations and subchannel-power allocation. We propose a joint subframe and bandwidth allocation scheme that can be jointly used with a downlink subchannel and power allocation scheme. Simulation results show that the performance of the proposed scheme is significantly better than that of [19].

3. SYSTEM MODEL

We consider a mobile multihop relay (MMR) system consisting of a BS and M RSs that are fixed. There are N mobile stations[†] (MSs), where each user is assigned to the station (BS or RS) of smallest distance. A sample MMR cell is shown in Figure 1. In this example there are six user. Two of them are

[†]In the simulations we keep the MS locations fixed but simulate the effects of mobility through Rayleigh fading and log-normal shadowing.

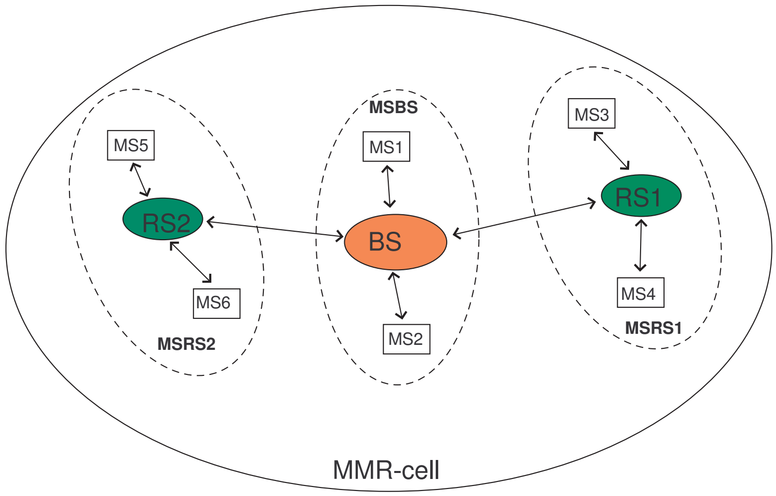

Figure 1. Topology of a MMR cell with a BS and two relay stations (RS_1 and RS_2). The BS is serving the MSs in the set MS_{BS} directly (MS_1 and MS_2). Two relay stations (RS_1, RS_2) are used to enhance the system throughput of BS and serve MSs in the set MS_{RS1} (MS_3, MS_4) and MS_{RS2} (MS_5, MS_6). The MMR cell includes the coverage area of the BS and all the RSs.

assigned to the BS and the rest are assigned to RSs. Let MS_{BS} and MS_{RS} denote the MSs assigned to the BS and any of the RSs , respectively. We consider frame by frame resource allocation. A frame is of duration T_f and it is divided into two subframes as in Figure 2. In the first subframe the transmissions from the BS occur (BS-RS and BS-MS). In the second subframe the transmissions from the RSs occur. Each subframe is further shared horizontally by the users. We consider an OFDMA system where a total bandwidth of W is divided into K subchannels. The transmissions of all links in a subframe occur simultaneously but in disjoint subchannels. We assume that the transmissions experience path loss, Rayleigh fading and log-normal shadowing. Channel conditions are assumed to be constant during a frame. Rayleigh fading is assumed to be flat in each subchannel and i.i.d for different users and subchannels. For the system and transmission quantities like channel condition, power and rate we define a superscript ϕ that becomes BS or RS depending on which link is intended (access or relay). Let the spectral efficiency achieved by user n at subchannel k be $S_n^{\phi}(k) = \log_2\left(1 + p_n^{\phi}(k)c_n^{\phi}(k)\right)$, where $\phi = BS, RS$. Here $p_n^{\phi}(k)$ is the transmission power allocated to user n at subchannel k and link ϕ. Parameter $c_n^{\phi}(k)$ is the channel condition $c_n^{\phi}(k) = \frac{\beta h_n^{\phi}(k)}{N_0 W/K}$, where $h_n^{\phi}(k)$ is the combined channel gain and $N_0 W/K$ is the noise power at a subchannel of bandwidth W/K Hz. The parameter β is the SNR-gap. In the 802.16 standard certain modulation and coding pairs are defined and these pairs can be used when the signal to noise ratio (SNR) is greater then corresponding threshold values [17]. Using a coefficient $\beta < 1$ (typically $\beta = 0.25$) fit the above Shannon capacity formula to this SNR threshold-rate relation in the standard.

We assume centralized scheduling and assume that the BS can perfectly obtain the channel condition parameters of all RSs and MSs.

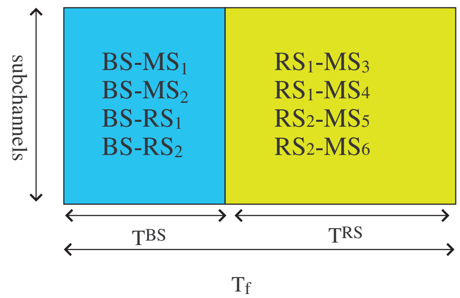

Figure 2. An MMR frame is divided into two subframes, which are called BS and RS subframes. Subframes are further divided to users horizontally, where a link transmits in using certain set of subchannels throughout the subframe duration.

4. POWER MINIMIZATION SUBJECT TO RATE CONSTRAINTS

Our aim in this chapter is to satisfy rate requirement of each link by using minimum total weighted power. The weights are especially important in the uplink case. The users at the cell edge are likely to cause intercell interference and their weights can be set higher to decrease their transmission power.In the downlink, the weights of the BS-RS, BS-MS and MS-RS transmissions can be chosen to adjust power expenditure by the BS and RS. Since the RSs are deployed closer to the cell edge their transmission power must be adjusted to limit interference to other cells.

Let R_n^{ϕ} be the target rate of user n at link ϕ in bits per subchannel. The parameters to be optimized are the subframe durations T^{BS}, T^{RS} and the power values. For user n and subchannel k, let $p_n^{BS}(k)$ and $p_n^{RS}(k)$ be the power values for the BS-RS and RS-MS links. The objective is to minimize the weighted total power of all link power expenditures, where α_n^{ϕ} is the link weight for user and and subframe ϕ. Resource allocation constraints are the subframe duration and the constraint that each subchannel to be used by at most one transmission. We formulate the power minimization problem as follows:

$$\min_{T^{RS},\mathbf{p^{BS}},\mathbf{p^{RS}}} \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^{BS} p_n^{BS}(k) + \alpha_n^{RS} p_n^{RS}(k) \qquad (1)$$

s.t

$$T^{\phi} \sum_{k=1}^{N} \log_2\left(1 + p_n^{\phi}(n) c_n^{\phi}(k)\right) \quad \geq \quad R_n^{\phi},\ \phi = BS, RS, n = 1,\ldots,N \qquad (2)$$

$$\mathbf{p}^{\phi} \quad \in \quad \mathcal{D},\ \phi = BS, RS \qquad (3)$$

where $T^{BS} = T_f - T^{RS}$. Here $\mathcal{D}$ is the set of power allocations such that for all $k = 1,\ldots,K$ and $\phi = BS, RS$, $p_n^{\phi}(k) > 0$ for only one user in $n = 1,\ldots,N$ [12]. We solve this problem by using Lagrange dual decomposition method. We can write the Lagrange dual as follows:

$$L(\mathbf{p},\overline{\mu},T^{RS}) = \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^{BS} p_n^{BS}(k) + \alpha_n^{RS} p_n^{RS}(k) - \sum_{\phi=BS,RS}\sum_{n=1}^{N} \mu_n^{\phi}(T^{\phi} \sum_{k=1}^{K} \log_2\left(1 + p_n^{\phi}(k) c_n^{\phi}(k)\right) - R_n^{\phi}) \quad (4)$$

where $\overline{\mu} = \{\mu_n^{\phi},\ \phi = BS, RS\ n = 1,\ldots,N\}$. For a given relay subframe time T^{RS}, the problem reduces to minimum power allocation in [12]. By taking derivative with respect to $p_n^{\phi}(k)$, we obtain the following expression for transmission power

$$\frac{\partial L}{\partial p_n^{\phi}(k)} = 0 \Rightarrow p_n^{\phi}(k) = \left(\frac{\mu_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}} - \frac{1}{c_n^{\phi}(k)}\right)^+, \forall n, k, \phi \tag{5}$$

Here the operator $(x)^+$ means $\max(0,x)$. Using (5) in (4), we obtain the Lagrangian in terms of Lagrange multipliers $\mu_n(k), \forall n, k$ and $T^{\phi}, \phi = BS, RS$.

$$L(\mathbf{p}, \mu, T^{RS}) = \sum_{\phi=BS,RS} \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_n^{\phi} \left(\frac{\mu_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}} - \frac{1}{c_n^{\phi}(k)}\right)^+ - \sum_{\phi=BS,RS} \sum_{n=1}^{N} \mu_n^{\phi} T^{\phi} \sum_{k=1}^{K} \left(\log_2\left(\frac{\mu_n^{\phi} c_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}}\right)\right)^+ + \sum_{\phi=RS,BS} \sum_{n=1}^{N} \mu_n^{\phi} R_n^{\phi} \tag{6}$$

For a given T^{RS}, minimization of the Lagrange dual above can be decomposed into independent problems for each subcarrier and subframe. Each subcarrier in a subframe has to be occupied by at most one user. The optimal user that occupies subcarrier k given T^{RS} Lagrange multipliers $\overline{\mu}$ is denoted by $A_k(\overline{\mu}, T^{RS})$ and found as [12],

$$A_k^{\phi}(\overline{\mu}, T^{RS}) = \arg\min_n \left\{ \alpha_n^{\phi} \left(\frac{\mu_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}} - \frac{1}{c_n^{\phi}(k)}\right)^+ - \mu_n^{\phi} T^{\phi} \left(\log_2\left(\frac{\mu_n^{\phi} T^{\phi} c_n^{\phi}}{\ln 2\alpha_n^{\phi}}\right)\right)^+ \right\}, \tag{7}$$

for $k = 1,\ldots,K, \phi = BS, RS$. At each subcarrier the minimizing user in (7) is the transmitting user. For a given T^{RS} value we can find the optimal vector $\mu_n^{\phi}, \phi = BS, RS; n = 1,\ldots,N$ using the Ellipsoid method [20]. In this method, first an initial ellipsoid covering the optimal solution is found. Then using a suitable subgradient this ellipsoid is narrowed down step by step, until convergence. The volume of the ellipsoid gets smaller at each step. As in [12], $d_n^{\phi} = \frac{R_n^{\phi}}{T^{\phi}} - \sum_{k=1}^{K} \log_2(1 + p_n^{\phi *}(k) c_n^{\phi}(k))$ is a suitable choice for the subgradient. However, we also need to take into account the constraint $\mu_n^{\phi} \geq 0, \forall n$. At each step we check if this constraint is satisfied. If not, we choose the subgradient $d_n^{\phi} = -(-\mu_n^{\phi})^+$. If yes, we use the former subgradient. The details of implementation can be found in Appendix B. For a given T^{RS} the optimal Lagrange dual (minimum total power) becomes,

$$L(\mu^*, T^{RS}) = \sum_{k=1}^{K} \alpha_{k*}^{BS} \left(\frac{\mu_{k*}^{BS}(T_f - T^{RS})}{\ln 2\alpha_{k*}^{BS}} - \frac{1}{c_{k*}^{BS}(k)}\right)^+ + \alpha_{k*}^{RS} \left(\frac{\mu_{k*}^{RS}(T^{RS})}{\ln 2\alpha_{k*}^{RS}} - \frac{1}{c_{k*}^{RS}(k)}\right)^+ \tag{8}$$

where k^* is the minimizing user in (7).

Proposition 1. *Optimal T^{RS} is obtained if the following equality is satisfied:*

$$\sum_{n=1} \frac{\mu_n^{BS} R_n^{BS}}{T_f - T^{RS*}} = \sum_{n=1} \frac{\mu_n^{RS} R_n^{RS}}{T^{RS*}} \tag{9}$$

The optimal T^{RS*} is unique and can be found using bisection on $[0, T_f]$.

Proof 1. *In Appendix A*

Hence the optimal subframe duration, subchannel allocations and power can be found using bisection and at each step of the bisection, ellipsoid method is applied. During the bisection subframe allocation and subchannel allocation always stay in the feasible region at each step, that is, $T^{RS} + T^{BS} = T_f$ and $\overline{\Omega} \in \mathcal{D}$ at each step. This gives us the chance to stop the algorithm at any iteration and find the optimal power allocation that satisfies the rate constraints for given subframe and subchannel allocation. This can be done in order to speed up the algorithm and cut the computational cost. Another issue is that the subframe length must be an integer number of time slot length. In order to satisfy this requirement, we need to round the subframe length and then perform another round of optimization for subchannels and powers. Another solution to find the optimal subframe length is exhaustive search in $[1, \frac{T_f}{T_s} - 1]$, where T_s is the time slot length. This has a complexity of $O(T_f/T_s)$, while binary search has complexity $O(\log_2 T_f/T_s)$. So our solution decreases the complexity significantly as the number of slots per frame increases.

5. SUBOPTIMAL SUBFRAME AND BANDWIDTH ALLOCATION

Finding the optimal subframe allocation requires binary search and application of ellipsoid method at each search step. Ellipsoid method is a robust and successful method, however it has a very slow convergence (takes $O(N^2)$ iterations to converge , where N is the number of users). Therefore it can be prohibitively complex and a much less complex, yet successful suboptimal solution is needed. In order to reduce the computational complexity, we first divide the problem into two subproblems. In the first subproblem we propose a subframe and bandwidth allocation algorithm, where we use the averaged user channel conditions $\overline{c}_n^{\phi}$, where the average is taken over all subchannel channel conditions $c_n^{\phi}(k)$. Then we simplify resource allocation by assuming bandwidth as a continuously divisible quantity [17] and convert the problem from discrete subchannel allocation to continuous bandwidth allocation. As a result of solving this subproblem we obtain subframe durations and decouple two subframes. We also obtain bandwidth values and in the second subproblem, using these bandwidth vales (i.e. number of subchannels) we perform subchannel allocation waterfilling-based power allocation. Let w_n^{ϕ} and p_n^{ϕ} be the number of subchannels and power allocated to user n at link $\phi = BS, RS$. The first subproblem is formulated as follows,

$$\max \sum_{\phi=BS,RS} \sum_{n=1}^{N} -\alpha_n^{\phi} p_n^{\phi} \tag{10}$$

s.t.

$$T^{\phi} w_n^{\phi} \log_2\left(1 + \frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right) \geq R_n^{\phi},\ \phi = BS, RS,\ n = 1, \ldots, N \tag{11}$$

$$\sum_{n=1}^{N} w_n^{\phi} \leq K \tag{12}$$

$$T^{RS} + T^{BS} \leq T_f \tag{13}$$

This problem has a concave objective function with convex constraint set. The Lagrange dual is,

$$L(\mathbf{p},\mathbf{w},\overline{\lambda_w},\overline{\lambda}) = \sum_{\phi=BS,RS}\left(\sum_{n=1}^{N} -\alpha_n^{\phi} p_n^{\phi} + \lambda_w^{\phi}\left(K - \sum_{n=1}^{N} w_n^{\phi}\right)\right) + \lambda_T(T - T^{RS} - T^{BS})$$
$$+ \sum_{\phi=BS,RS}\sum_{n=1}^{N}\lambda_n^{\phi}\left(T^{\phi} w_n^{\phi}\log_2\left(1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right) - R_n\right) \quad (14)$$

Using the equations $\frac{\partial L}{\partial p_n^{\phi}} = 0$ and $\frac{\partial L}{\partial w_n^{\phi}} = 0$ we obtain,

$$\lambda_n^{\phi} = \frac{\alpha_n^{\phi}\ln 2}{T^{\phi}\overline{c}_n^{\phi}}\left(1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right),\ \forall n \quad (15)$$

$$\frac{1}{\lambda_n^{\phi}} = \frac{T^{\phi}}{\ln 2\lambda_w^{\phi}}\left[\ln\left(1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right) - \frac{\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}}{1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}}\right],\ \forall n \quad (16)$$

respectively. Multiplying (15) and (16), we can eliminate λ_n^{ϕ} and obtain the following:

$$\frac{\lambda_w^{\phi}\overline{c}_n^{\phi}}{\alpha_n^{\phi}} = f(\gamma_n^{\phi}) = (1+\gamma_n^{\phi})\ln(1+\gamma_n^{\phi}) - \gamma_n^{\phi} \quad (17)$$

where $\gamma_n^{\phi} = \frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}$ is the SNR for user n at link ϕ. Function f is monotonic increasing and $\log_2\left(1+f^{-1}\left(\lambda_w^{\phi}\overline{c}_n^{\phi}/\alpha_n^{\phi}\right)\right)$ gives the spectral efficiency that can be achieved by user n at subchannel k corresponding to Lagrange multiplier λ_w^{ϕ}. Using $\frac{\partial L}{\partial T^{\phi}} = 0$ we obtain,

$$\lambda_T = \sum_{n=1}^{N}\lambda_n^{\phi} w_n^{\phi}\log_2\left(1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right) \quad (18)$$

$$T^{\phi} = \sum_{n=1}^{N}\frac{\lambda_n^{\phi} R_n^{\phi}}{\lambda_T} \quad (19)$$

$$T^{\phi} = \sum_{n=1}^{N}\frac{\ln 2\alpha_n^{\phi}\left(1+\frac{p_n^{\phi}\overline{c}_n^{\phi}}{w_n^{\phi}}\right)R_n^{\phi}}{T^{\phi}\lambda_T\overline{c}_n^{\phi}} \quad (20)$$

$$T^{\phi} = \sqrt{\sum_{n=1}^{N}\frac{\ln 2\alpha_n^{\phi}\left(1+f^{-1}\left(\frac{\lambda_w^{\phi}\overline{c}_n^{\phi}}{\alpha_n^{\phi}}\right)\right)R_n}{\lambda_T\overline{c}_n^{\phi}}} = f_T^1(\lambda_T,\lambda_w^{\phi}) \quad (21)$$

Equation (19) is obtained by replacing $w_n^{\phi}\log_2(1+p_n^{\phi}\overline{c}_n^{\phi}/w_n^{\phi})$ with R_n^{ϕ}/T^{ϕ}. Equation (20) is obtained by replacing λ_n^{ϕ} with (15). Lastly, (21) is obtained by replacing $p_n^{\phi}\overline{c}_n^{\phi}/w_n^{\phi}$ by $f^{-1}(\lambda_w^{\phi}\overline{c}_n^{\phi}/\alpha_n^{\phi})$ rearranging the expression.

Using the total bandwidth constraint we obtain a second equation for T^{ϕ}:

$$T^{\phi} = \sum_{n=1}^{N}\frac{R_n}{\log_2\left(1+f^{-1}\left(\frac{\lambda_w^{\phi}\overline{c}_n^{\phi}}{\alpha_n^{\phi}}\right)\right)K} = f_T^2(\lambda_w^{\phi}) \quad (22)$$

We obtained two functions that relate λ_T and λ_w^ϕ in (21) and (22). The function $f_T^1(\lambda_T,\lambda_w^\phi)$ is increasing in λ_w^ϕ, while $f_T^2(\lambda_w^\phi)$ is decreasing. Therefore there is a unique λ_w^ϕ for a given λ_T that satisfies $f_T^1(\lambda_w^\phi,\lambda_T)=f_T^2(\lambda_w^\phi)$. As λ_T increases, the T^ϕ value at the intersection $f_T^1(\lambda_w^\phi,\lambda_T)=f_T^2(\lambda_w^\phi)$ increases. Because of this monotonicity property the optimal Lagrange multipliers can be found using bisection method. For the optimal $\lambda_w^{\phi *}$, optimal bandwidth and power values can be found simply by

$$w_n^{\phi *} = \frac{R_n}{T^\phi \log_2\left(1+f^{-1}\left(\frac{\lambda_w^{\phi *}\overline{c}_n^\phi}{\alpha_n^\phi}\right)\right)} \quad (23)$$

$$p_n^{\phi *} = f^{-1}\left(\frac{\lambda_w^{\phi *}\overline{c}_n^\phi}{\alpha_n^\phi}\right) w_n^{\phi *}\overline{c}_n^\phi,\ \forall n, \phi = BS, RS \quad (24)$$

We call this scheme as Joint Subframe-Bandwidth-Power Scheme (JSBP). It has a complexity of $O(N)$ in terms of number of users and its complexity does not depend on the number of subchannels and number of slots in a frame. After finding the subframe times T^{BS*} and T^{RS*}, the problem reduces to two separate multiuser OFDMA power minimization problems, which can be solved using the ellipsoid method in [12]. In order to further simplify the problem, heuristics can be proposed to allocate subchannels as in the next section.

5.1. Subchannel and Power Allocation

The JSBP scheme proposed in the previous section produces bandwidth allocations $(\mathbf{w}^{BS},\mathbf{w}^{RS})$ and powers $(\mathbf{p}^{BS},\mathbf{p}^{RS})$. Bandwidths that we found are not necessarily integer multiples of subchannel bandwidths. These bandwidth values can be quantized in order to find the number of subchannels $[w_n^\phi]$ allocated to each link. After quantization if $\sum_{n=1}^N [w_n^\phi] < K$, then at each step find the user maximizing $[w_n^\phi](2^{R_n^\phi/[w_n^\phi]T^\phi}-1)/\overline{c}_n^\phi(k)$(power) and allocate one more subchannel until equality is satisfied. If $\sum_{n=1}^N [w_n^\phi] > K$ then at each step find the user with $w_n^\phi > 1$ and maximizing $[w_n^\phi]-w_n^\phi$, and decrease its subchannels by one. We then recalculate the power requirement to satisfy those rates for each user as $p_n^{\phi'}$. As in [11] we have the number of subchannels and number of bits per symbol for each user. Hence the heuristic methods proposed in [11] can be used. One of the methods mentioned in this paper is Vogel's method.

For the sake of completeness we give the Vogel's algorithm with our notation. The following algorithm is run separately for $\phi = BS$ and $\phi = RS$ subframes. Before the algorithm is run, the operation $T^{RS} = \max(T_s, T_s \times [T^{RS}/T_s])$,$T^{BS} = T_f - T^{RS}$ is done to quantize subframe duration to integer number of time slots.

- Initialize $\Omega_k = 0$ for $k = 1,\ldots,K$, $\mathcal{U} = \{1,\ldots,N\}$
- Quantization: Round $w_n^\phi = [w_n^\phi], \phi = BS, RS, n = 1,\ldots,N$. Perform reallocations if $\sum_{n=1}^N w_n^\phi \neq K$‡
- Compute $p_n^{\phi'} = w_n^\phi(2^{R_n^\phi/w_n^\phi T^\phi}-1)/\overline{c}_n^\phi(k)$ for $n = 1,\ldots,N$, $k = 1,\ldots,K$§.

‡The reallocations after bandwidth quantization are performed as follows: If $\sum_{n=1}^N w_n^\phi < K$, then at each step find the user maximizing $w_n^\phi(2^{R_n^\phi/w_n^\phi T^\phi}-1)/\overline{c}_n^\phi(k)$(power) and allocate one more subchannel until equality is satisfied. If $\sum_{n=1}^N w_n^\phi > K$ then at each step find the user with $w_n^\phi > 1$ and minimizing $w_n^\phi(2^{R_n^\phi/w_n^\phi T^\phi}-1)/\overline{c}_n^\phi(k)$, and decrease its subchannels by one.

§Here wethe power requirement to satisfy those rates for each user as $p_n^{\phi'}$.

- Compute subchannel costs for each user as $\kappa_n^\phi(k) = \alpha_n^\phi p_n^{\phi'} \overline{c}_n^\phi(k)/c_n^\phi(k),\ \forall n,k$
- Calculate user penalties as $\pi_n^\phi = \kappa_n^{\phi(w_n^\phi+1)} - \kappa_n^{\phi(1)}$, Here $\kappa_n^{\phi(w_n^\phi+1)}$ means the $(w_n^\phi+1)$th smallest subchannel cost among $\{\kappa_n^\phi(k)|\Omega_k = 0\}$
- Repeat following two steps until $\Omega_k > 0, \forall k = 1,\ldots,K$:
  - Find $n^* = \arg\max_{n\in\mathcal{U}}\{\kappa_n^\phi\}$.
  - Find $k^* = \arg\min_{k|\Omega_k=0}\{\kappa_{n^*}^\phi(k)\}$
  - Set $\Omega_{k^*} = n^*$. Set $w_{n^*}^\phi = w_{n^*}^\phi - 1$. If $w_{n^*}^\phi = 0$ then update $\mathcal{U} = \mathcal{U} - \{n^*\}$
  - Update penalties for all users in the set $\mathcal{U}$.
- Solve the following problem for $\phi = BS, RS$ and $n = 1,\ldots,N$: $\min_{p_n^\phi(k)} \sum_{k:\Omega_k=n} p_n^\phi(k)$ s.t. $T^\phi \sum_{k:\Omega_k=n} \log_2(1 + p_n^\phi(k)c_n(k)) \geq R_n^\phi$. This can be solved using water-filling.

In the above algorithm user penalties are computed at each step. The user with the smallest penalty is chosen and that user is allocated the available subchannel with the smallest cost. This is a polynomial time algorithm that can be used in real-time OFDM systems [11].

5.2. NUMERICAL RESULTS

In this section we test and compare the performance of the optimal and suboptimal resource allocation algorithms. We consider the following algorithms,

1. Subframe Allocation

   - **JSBP**: This is the joint subchannel bandwidth power allocation scheme we proposed in Section 5.
   - **Muller**[19] : This subframe allocation scheme first determines a fixed number of bits per Hz for all links. Then it tries all possible number of slots for subframe durations and chooses the one that requires minimum power.
   - $\mathbf{T^{BS}} = \mathbf{T^{RS}}$: This scheme simply divides the frame into two and will be used as a benchmark.

2. Subchannel and Power Allocation:

   - **Vogel** [11]: This is the heuristic that was also used in [11] and explained in Section 5.1. If JSBP is used than number of subchannels and power values that are necessary for the Vogel's algorithm are already found. If Mueller's algorithm is used, then these values can be found by plugging the subframe durations in problem (10)-(12) and solving the problem using similar convex programming techniques.
   - **VogelDist**: This is our extension for Vogel's algorithm. Once the number of subchannels are computed, we know the total number of subchannels required for users belonging to each relay station. Then, each relay can perform its own allocations from its given set of subchannels. This limits the subchannel diversity, however it has the advantages of a distributed scheme; the RSs don't need to feedback channel conditions of each user and subchannel to the BS. They only need to feedback average values ($\overline{c}_n^{RS}$). Please note that allocations for the BS subframe is the same.

3. **Optimal** : This is jointly optimal subframe, subchannel and power allocation solution presented in Section 4. It has a slow convergence, which makes it difficult to implement in real-time, however, it will be considered as a benchmark.

We perform simulations for pairs of subframe and subchannel allocation algorithms listed above and perform them based on the parameters in Table I. In addition to this we will compare them also with the optimal solution presented in Section 4. We consider a tandem topology with cell size of 2000m with BS at the origin. Two relay stations are located at coordinates (-1400m, 0m) and (1400m, 0m). We assume that $R_n^{BS} = R_n^{RS}$ for $n \in MS_{RS1}$ and $n \in MS_{RS2}$.

Parameter	**Value**
Cell radius	2km
User Distances	0.4,0.8,1.2,1.6,2.0km
RS Distance	1.4km
# Relay stations (M)	2
W, K	1MHz, 128
Frame Length T_f	4 msec
Slot Length T_s	0.1 msec
AWGN p.s.d.(N_0)	-174dBm/Hz
BS-RS PL(d)(in dB)	$36.5 + 23.5\log_{10} d + \psi_{dB}^{BS-RS}$
RS-MS PL(d)(in dB)	$31.5 + 35\log_{10} d + \psi_{dB}^{RS-MS}$
BS-MS PL(d)(in dB)	$31.5 + 35\log_{10} d + \psi_{dB}^{BS-MS}$
$\psi_{dB}^{BS-MS}, \psi_{dB}^{RS-MS}$	$\sim N(0dB, 8dB)$
ψ_{dB}^{BS-RS}	$\sim N(0dB, 3.4dB)$
SNR gap coefficient (β)	0.25

Table I. Simulation Parameters

Figure 3 show the distributions of weighted total powers for the case of 4 users and and two relay stations. There are two users located at coordinates $(+2000, 0)$ and $(-2000, 0)$ meters and they are assigned to relays 1 and 2, respectively. There are two users at $(+400, 0)$ and $(-400, 0)$, which are assigned to the BS. Users have a total rate constraint of 3Mbps. For users assigned to relays stations we assume that $R_n = R_n^{RS} = R_n^{BS}$. We used the weights $\alpha_n^{RS} = 1.5, \alpha_n^{BS} = 0.5, \forall n$. We used higher weights for relay links because RSs can be more power limited and increasing their power can cause intercell interference as they are located close to the cell edge. We can observe from Figure 3 that Optimal is the best, as expected. We also observe that the performances of Muller and JSBP are quite close to that of Optimal. In the second subgraph we take a close look at the performances. We see that for this case JSBP is better than Muller. $T^{RS} = T^{BS}$ results in the worst performance.

Table II shows the mean weighted power expenditures for the BS and both stations for the same set of parameters. We see that using JSBP and Vogel results in almost optimal performance however, using $T^{RS} = T^{BS}$ results in fifty percent more energy expenditure.

Figure 4 shows the relation of total weighted power vs. rates for a system of 20 users. We located the users at coordinates (on the x-axis) $-2000, -1600, -1200, -800, -400, 400, 800, 1200, 1600, 2000$ meters. There are 2 users at each point. Users of distance $\mp 2000, \mp 1600, \mp 1200$ are assigned to the relays and the rest are assigned directly to the BS. Each user is assigned to the closest station. Optimal assignment of users to stations is a subject of future research. We increased the total system load from

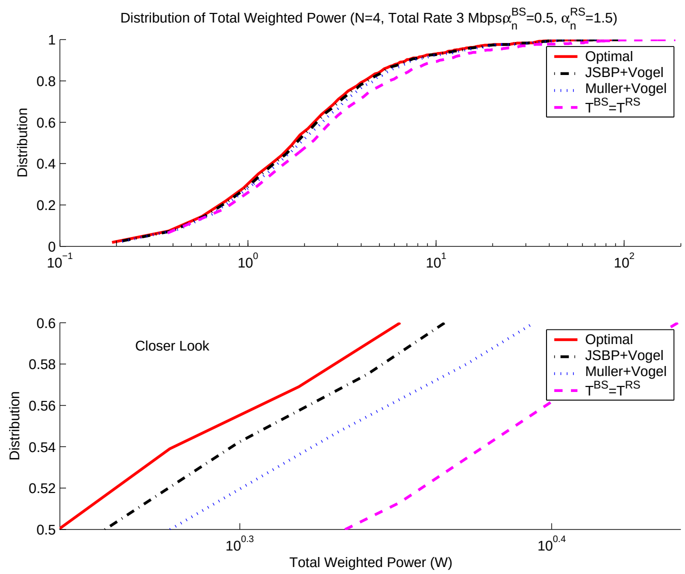

Figure 3. Total weighted power for three different algorithms (2 users and 2 relay stations).

	Optimal	JSBP	Muller	$T^{BS} = T^{RS}$
$\mathtt{mean}(P^{BS})$	0.78794	0.66583	0.52345	0.53426
$\mathtt{mean}(P^{RS})$	2.7581	3.0324	3.3662	4.8611

Table II. Mean BS and RS weighted total power for N=4

2 to 7 Mbps. Individual rates are generated randomly such that their total is equal to the total load. We observe that using our proposed JSBP jointly with Vogel's method results in best performance. It provides 10 percent improvement over Muller+Vogel. We also observe that even using VogelDist with JSBP performs equally with Muller+Vogel. Dividing the frame equally results in the worst performance.

In Figure 5 we observe the relation of total weighted power expenditure and number of users. For this purpose, we considered a total system load of 6 Mbps, and increased the number of users from 10 to 50. User locations are still at the discrete levels mentioned above for the Figure 4, and the ratio of users located at each coordinate point is equal. Station assignment to the users is the same as in Figure 4. User rates are generated randomly such that their sum is always 6Mbps, therefore as number of users increase the average rate per user decreases. We observe that JSBP scheme achieves 20 percent less power than Muller's scheme at high number of users. We observe that even using the distributed version of Vogel's method with JSBP results in better performance when $N = 50$. Another interesting observation is that power expenditure is non-monotonic in number of users. As N increases from 10-30, the multiuser diversity is in effect, hence power expenditure decreases. However as N increases from 30 to 50, most users begin to have only one or two subchannels, which makes the subchannel allocation more critical. Bandwidth allocation results in non-integer quantities and their quantization

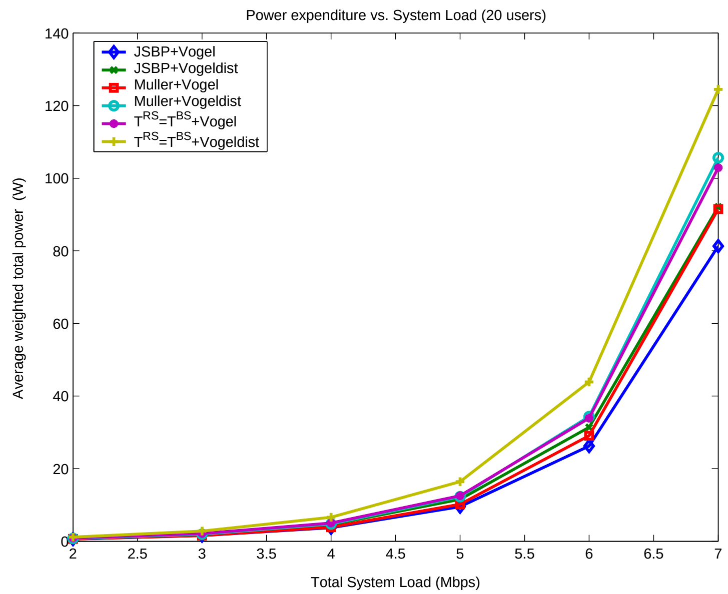

Figure 4. Average total power expenditures vs. user rates.

results in significant deviations in power requirements. If each user had high number of subchannels then water-filling would mitigate the effects of subchannel allocation. When each user has one or two channels then waterfilling doesn't have much or any effect.

6. FAIR RESOURCE ALLOCATION

In this section we consider the problem of fair resource allocation for delay tolerant data traffic. Fairness is very important and especially preferred to throughput as a criterion because in a metropolitan area network user channel conditions vary considerably. Throughput maximizing solutions provide little or no service to users at the cell edge. For a given power constraint, power optimization has limited effect on the performance, therefore we study the problem of fair subframe and subchannel allocation for constant power per subchannel. We redefine $c_n^{\phi}(k)$ including this power as $c_n^{\phi}(k) = \frac{\beta h_n^{\phi} P^{\phi}}{N_0 W}, \phi = BS, RS; n = 1, \ldots, N; k = 1, \ldots, K$. There a few papers in the literature that studies fair resource allocation in relay enhanced networks and [18] is one of them. We will briefly explain the proposed algorithm in [18] and propose an improvement for it. In [18] slot is divided *equally* into two subslots. In the first subslot the transmissions $BS \rightarrow MS_{BS}$ and $BS \rightarrow RS$ occur. In the second one the transmissions $RS \rightarrow MS_{RS}$ and $BS \rightarrow MS_{BS}$ occur. Hence, $BS \rightarrow MS_{BS}$ transmissions can occur in both time slots. First, the allocations in the second subslot occur, which is performed channel by channel.

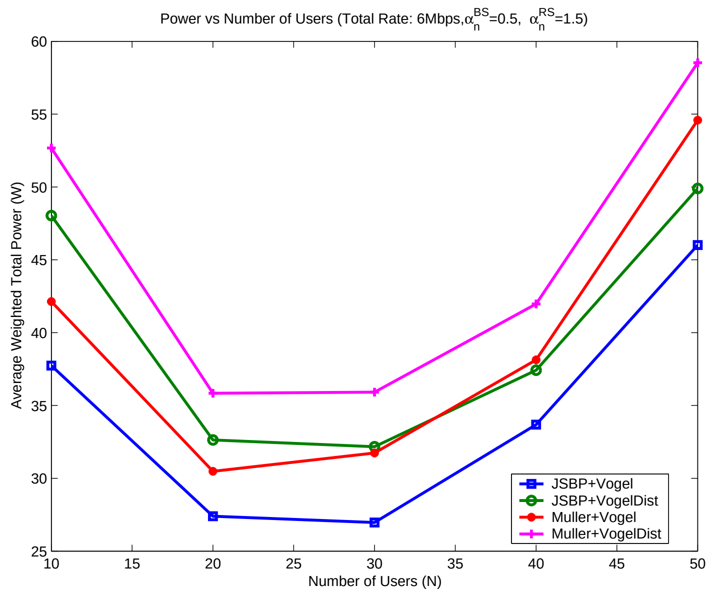

Figure 5. Total weighted power vs. Number of Users

For channel k each user has the following metric

$$\eta_{n,k} = \begin{cases} \frac{W/K \log_2\left(1+c_n^{BS}(k)\right)}{\alpha \overline{R}_n + (1-\alpha) r_n} & n \in MS_{BS} \\ \frac{W/K \log_2\left(1+c_n^{RS}(k)\right)}{\alpha \overline{R}_n + (1-\alpha) r_n} & n \notin MS_{BS} \end{cases} \tag{25}$$

where $\overline{R}_n$ is the time averaged rate received by user n, and r_n is the rate allocated to user n in the current slot and it is updated when a new subchannel is allocated to user n. Channels in the second subslot are allocated in a greedy manner based on (25).

The relay station that we consider (as well as in [18]) is a "prompt" relay, which means that the information transmitted from BS to RS is immediately decoded and retransmitted from RS to respective MSs. The effective data rate is the minimum of the BS-RS and RS-MS rates. Therefore in the first subslot sufficient resources should be allocated for each RS. The rest of the resources are shared among $BS \rightarrow MS_{BS}$ transmissions again according to metric (25).

This algorithm can be improved in a number of ways. 1) Current frame format of IEEE 802.16j is divided into BS and RS subframes instead of dividing into slots. 2) BS and RS only transmit in their own subframes as in our frame format in Figure 2. 3) Performance can be improved by adjusting the subframe durations T^{RS} and T^{BS}. 4) Algorithm in [18] requires feeding back the channel conditions of all users at all subchannels to the BS, which can be excessive. RSs have to feed back $N \times K$ quantities in total. Considering these issues we develop a method that determines the subframe times and amount of bandwidth allocated to each RS. As in the previous power minimization problem we average out the channel conditions of each user over all the subchannels and define $S_n^{\phi} = \log_2\left(1+\overline{c}_n^{\phi}\right), \phi = BS, RS$. We assume bandwidth as a continuously divisible quantity. Then we formulate the following proportional fair bandwidth allocation problem. Since the power is uniformly distributed, time-frequency product $T^{\phi}w_n^{\phi}$ can be defined as a "resource" b_n^{ϕ}. The total used resources has to be smaller than the bandwidth-frame product $W \times T_f$,

$$\max_{\mathbf{b}} \sum_n \log\left(\alpha R_n + (1-\alpha) r_n\right) \tag{26}$$

$$\begin{aligned} b_n^{BS} S_n^{BS} &\geq r_n, \; \forall n & (27)\\ b_n^{RS} S_n^{RS} &\geq r_n, \; \forall n \notin MS_{BS} & (28)\\ \sum_{\forall n} b_n^{BS} + \sum_{n \notin MS_{BS}} b_n^{RS} &\leq W T_f & (29) \end{aligned}$$

Solving this convex optimization problem using standard methods we obtain the following relations,

$$\begin{aligned} b_n^{RS}(\lambda_b) &= \left[\frac{1}{\lambda_b (1 + \frac{S_n^{RS}}{S_n^{BS}})} - \frac{\widetilde{\alpha} R_n}{S_n^{RS}}\right]^+, \; \forall \notin MS_{BS} & (30)\\ b_n^{BS}(\lambda_b) &= \left[\frac{1}{\lambda_b (1 + \frac{S_n^{BS}}{S_n^{RS}})} - \frac{\widetilde{\alpha} R_n}{S_n^{BS}}\right]^+, \; \forall \notin MS_{BS} & (31)\\ & & (32) \end{aligned}$$

For $n \in MS_{BS}$ we find $b_n^{BS}(\lambda_b) = \left[\frac{1}{\lambda_b} - \frac{\widetilde{\alpha} R_n}{S_n^{BS}}\right]^+$. We find the optimal λ_b^* such that (29) is satisfied. Then we find the subframe durations as $T^{BS} = \frac{1}{W}\sum_{\forall n} b_n^{BS}$ and $T^{RS} = \frac{1}{W}\sum_{n \notin MS_{BS}} b_n^{RS}$. We need to quantize these values to integer multiples of time slots. Then the bandwidth allocated to each RS is calculated as $W_m^{RS} = W\left(\sum_{n \in MS_{RS_m}} b_n^{RS}\right) / \left(\sum_{n \notin MS_{BS}} b_n^{RS}\right), \; \forall m = 1, \ldots, M$. These bandwidth values also need to be quantized to integer multiples of subchannel bandwidth. Then each RS is given that many subchannels and allocates these subchannels itself to its users according to the metric in (25). Users $n \in MS_{BS}$ are not served in the RS subframe. Then in the BS subframe BS first allocates enough subchannels for the RSs to support the rates allocated to RS users. The remaining subchannels are allocated to users MS_{BS} according to metric (25). Let's denote this algorithm as **Algorithm A**. This algorithm is semi-distributed since an RS can make its own allocations once it is given a set of subchannels. The RSs have to only feed back N quantities for the average user channel conditions and then N quantities for the allocated rates to users so that BS can allocate enough subchannels to each RS and transmit enough information for each user in those subchannels. As a benchmark, we propose another centralized algorithm, wherein all allocation is again performed by the BS according to the metric (25) but the subframe durations are still calculated using our proposed method. We call this **Algorithm B**.

6.1. Numerical Results

Figure 6 shows the comparison of simulation results for the three algorithms (Algorithms A, B and [18]) in terms of the total average throughput vs. number of users. We performed the simulations for $W = 1MHz$, $K = 128$ and $\alpha = 0.99$ as in [18]. In the BS subframe 20 W power and in RS subframe 10 W power is uniformly distributed to each subchannel. Here we make a *saturated queue* assumption, that is, we assume that there are always unlimited number of bits in the queues of the BS. This is an assumption that is made in many of the related works (including [18]) that study maximization of total throughput or proportional fairness. We see that despite Algorithm B requires less feedback than [18] it can provide significant improvement. We also see that Algorithm B achieves the best performance, which is expected because BS can use the complete set of subchannels, while in Algorithm A each RS can only use its allocated set of subchannels.

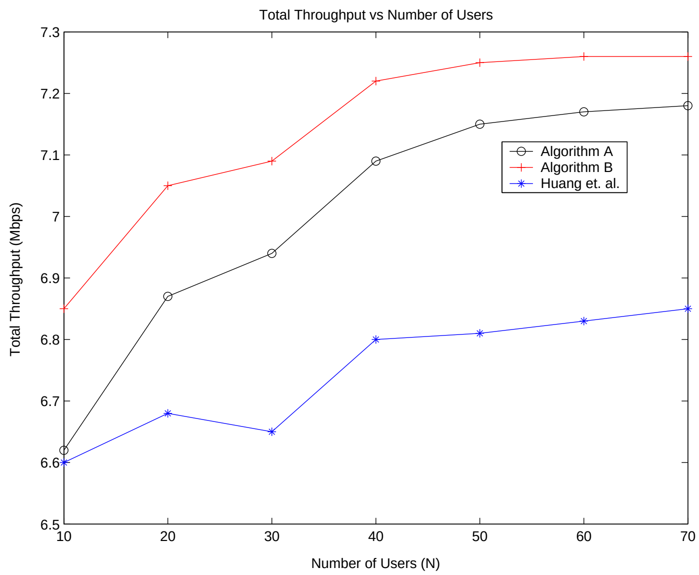

Figure 6. Total throughput vs. number of users for the three algorithms.

	$N=20$	$N=40$	$N=60$
Algorithm A	129.3	231.8	324.1
Algorithm B	129.8	232.5	324.9
Huang et. al.	128.9	231.0	321.7

Table III. Sum of logarithms of average user throughputs vs. N

Table III compares the sum of logarithms of average user throughputs, which is a measure of proportional fairness. We again see that Algorithms A and B have better fairness performance than [18]. We understand that the proposed algorithms achieve better throughput without sacrificing fairness.

7. CONCLUSIONS

In this paper we studied OFDMA based resource allocation in cellular systems with relays. Resource allocation involves optimally dividing the frame into BS and RS subframes, allocating subchannels to individual transmissions and loading each subchannel with optimum power. We considered two problems, which are minimizing power subject to rate constraints and maximizing fairness subject to power constraints. In the first problem we found the optimal solution and also proposed a suboptimal solution that performs close to the optimal. For the second problem we considered a recently proposed algorithm as a benchmark and proposed a subframe and subcarrier allocation algorithm that outperforms it.

The results of this research is applicable to mobile multihop relay (MMR) networks that are recently being standardized as IEEE 802.16j. Future work should include networks with heterogeneous traffic requirements, which requires joint solution of these two problems. Exploiting cooperative diversity is also made possible in the IEEE standard and is a subject of future work.

8. APPENDIX A: PROOF OF LEMMA 1

Optimal T^*_{RS} is obtained if $\frac{\partial L(\mathbf{p}^*,\overline{\mu^*},T^{RS})}{\partial T_{RS}} = 0$. Suppose that we change T_{RS} as $T_{RS}+\Delta_T$ and T^{BS} as $T^{BS}-\Delta_T$ so that total time constraint is still satisfied.

Given Δ_T we will find the new set of Lagrange multipliers $\mu_n^{\phi}+\Delta_{\mu,n}^{\phi},\ \forall n,\phi$ so that rate constraints are satisfied.

$$\begin{aligned}
R_n^{RS} &= (T^{RS}+\Delta_T)\sum_{k\in S_n}\log_2\left(\frac{(\mu_n^{RS}+\Delta_{\mu,n}^{RS})(T^{RS}+\Delta_T)c_n^{RS}(k)}{\ln 2\alpha_n^{RS}}\right)\\
0 &\cong \left[\Delta_T\sum_{k\in S_n}\log_2\left(\frac{\mu_n^{RS}T^{RS}c_n^{RS}(k)}{\ln 2\alpha_n^{RS}}\right)\right]+\frac{|S_n^{RS}|}{\ln 2}\frac{\mu_n^{RS}\Delta_T+\Delta_{\mu,n}^{RS}T^{RS}}{\mu_n^{RS}}\\
\Delta_{\mu,n}^{RS} &\cong -\frac{\mu_n^{RS}\Delta_T}{T^{RS}}\left(1+\frac{\ln 2}{|S_n^{RS}|}\sum_{k\in S_n}\log_2\left(\frac{\mu_n^{RS}T^{RS}c_n^{RS}(k)}{\ln 2\alpha_n^{RS}}\right)\right)\\
&= -\frac{\mu_n^{RS}\Delta_T}{T^{RS}}\left(1+\frac{\ln 2}{|S_n^{RS}|}\frac{R_n^{RS}}{T^{RS}}\right)
\end{aligned}\tag{33}$$

Here we assume that $\Delta_{\mu,n},\ \Delta_T$ goes to zero. Similarly as (33) we can find $\Delta_{\mu,n}^{BS}=\frac{\mu_n^{BS}\Delta_T}{T^{BS}}\left(1+\frac{\ln 2}{|S_n^{BS}|}\frac{R_n^{BS}}{T^{BS}}\right)$. Using these expressions we can write the change in total weighted power as a function of Δ_T.

$$\begin{aligned}
L(\mathbf{p}^*,\overline{\mu^*},T^{RS}+\Delta_T) = &\sum_{k=1:p_{k*}^{BS}(k)>0}^{K}\left(\frac{(\mu_{k*}^{BS}+\Delta_{\mu,k^*}^{BS})(T_f-T^{RS}-\Delta_T)}{\ln 2}-\frac{\alpha_{k*}^{BS}}{c_{k*}^{BS}(k)}\right)\\
&+\sum_{k=1:p_{k*}^{RS}(k)>0}^{K}\left(\frac{(\mu_{k*}^{RS}+\Delta_{\mu,k^*}^{RS})(T^{RS}+\Delta_T)}{\ln 2}-\frac{\alpha_{k*}^{RS}}{c_{k*}^{RS}(k)}\right)
\end{aligned}\tag{34}$$

Using $\Delta_{\mu,n}^{BS}=\frac{\mu_n^{BS}\Delta_T}{T^{BS}}\left(1+\frac{R_n^{BS}}{T^{BS}}\right)$, we can find,

$$\begin{aligned}
L(\mathbf{p}^*,\mu^*,T^{RS}+\Delta_T)-L(\mathbf{p}^*,\mu^*,T^{RS}) &= \Delta_T\left(\sum_{k:p_{k*}^{BS}(k)>0}^{K}\frac{R_{k^*}^{BS}\mu_{k^*}^{BS}}{|S_{k^*}^{BS}|(T_f-T^{RS})}-\sum_{k:p_{k*}^{BS}(k)>0}^{K}\frac{R_{k^*}^{RS}\mu_{k^*}^{RS}}{|S_{k^*}^{BS}|T^{RS}}\right)\\
\frac{L(\mathbf{p}^*,\mu^*,T^{RS}+\Delta_T)-L(\mathbf{p}^*,\mu^*,T^{RS})}{\Delta_T} &= \sum_{n=1}^{N}\frac{R_n^{BS}\mu_n^{BS}}{T_f-T^{RS}}-\sum_{n=1}^{N}\frac{R_n^{RS}\mu_n^{RS}}{T^{RS}}
\end{aligned}\tag{35}$$

So, if $\sum_{n=1}^{N} \frac{R_n^{BS}\mu_n^{BS}}{T_f - T^{RS}} > \sum_{n=1}^{N} \frac{R_n^{RS}\mu_n^{RS}}{T^{RS}}$, then the total power increases with increasing T^{RS}. If the inequality is reverse, then total power is a decreasing function of T^{RS}. Then there exists an optimal T^{RS*} such that the total power is minimum. If we use (33), we see that $\frac{R_n^{BS}\mu_n^{BS}}{T_f - T^{RS}}$ is monotonic increasing and $\frac{R_n^{RS}\mu_n^{RS}}{T^{RS}}$ is monotonic decreasing for all n. So, The equality occurs at a single point and that can be found by using bisection on $[0, T_f]$. Figure 7 illustrates this fact. In this set of simulations we considered a system of 10 users located at -2000, -1600, -1200, ..., +1600,+2000 meters. We consider a single channel instant and vary T^{RS} (hence also T^{BS}) and optimize total weighted power for each value. We see that minimum in the first graph occurs at the intersection in the second graph. We also observe the monotonicity of $\sum_{n=1}^{N} \frac{R_n^{BS}\mu_n^{BS}}{T_f - T^{RS}}$ and $\sum_{n=1}^{N} \frac{R_n^{RS}\mu_n^{RS}}{T^{RS}}$ as a function of T^{RS}.

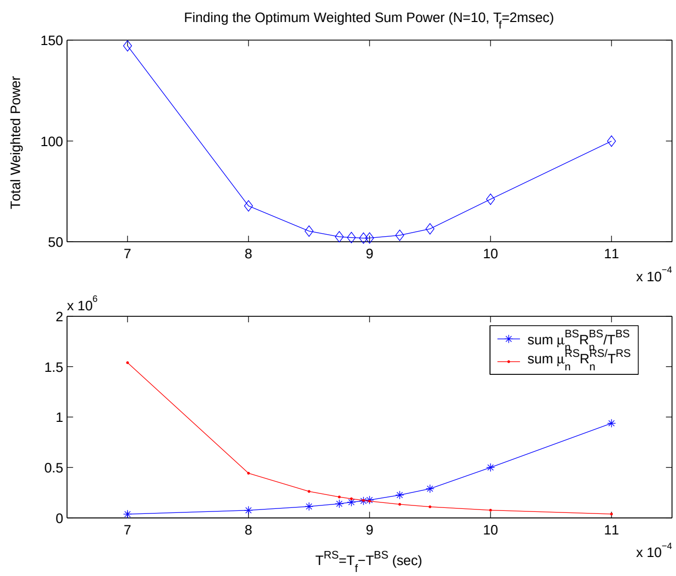

Figure 7. Weighted sum power reaches minimum when $\sum_{n=1}^{N} \frac{R_n^{BS}\mu_n^{BS}}{T_f - T^{RS}} = \sum_{n=1}^{N} \frac{R_n^{RS}\mu_n^{RS}}{T^{RS}}$.

9. Appendix B: Implementation of the Ellipsoid Method

Ellipsoid method can be used to find the $\mu_n^{\phi}, n = 1, \ldots, N, \phi = BS, RS$ that minimizes $L(\mathbf{p}, \overline{\mu}, T^{RS})$ in (34). Given T^{RS} and $T^{BS} = T_f - T^{RS}$ the problem can be separated into BS and RS subproblems. Using ellipsoid method we localize the optimal $\overline{\mu}^{\phi*} = \{\mu_1^{\phi*}, \ldots, \mu_N^{\phi*}\}$ inside an ellipsoid. In this section we will explain the implementation of the ellipsoid method for this problem. This method was used in [12] but was not explained in every detail. We start with an ellipsoid $\varepsilon^0 = \{z \in R^N : (\mathbf{x} - \overline{\mu}^{\phi,0})^T P_0^{-1} (x - \overline{\mu}^{\phi,0})\} \leq 1$. In order to find this initial ellipsoid, we need to find a hypercube that is guaranteed to include the optimal solution, i.e. a_n^{ϕ} such that $0 \leq \overline{\mu}^{\phi,*} \leq a_n^{\phi}, \forall n$. From now on we will remove the superscript ϕ. For user n the upper bound will be denoted as $2\overline{\mu}^{\phi,0}$ and it will be divided into two to find the center of the hypercube (and ellipsoid). In order to find the upperbound for user i we first determine any subchannel allocation that allocates at least one subchannel to a user with non zero rate. Let $S_k = n$ if subchannel k is allocated to MS n. Then we find the optimal power allocation for each user that satisfies the rate constraints below,

$$\sum_{S_k=j} T\log_2(1+p_j(k)c_j(k)) = R_j,\ \forall j \neq i \tag{36}$$

$$\sum_{S_k=j} T\log_2(1+p_j(k)c_j(k)) = R_j+1,\ j = i, \tag{37}$$

Now, consider the Lagrange dual function at point x^* (optimal dual variable):

$$g(x^*) = \min L(\mathbf{p}^*, x^*) \tag{38}$$

Since the subchannel allocation that was assumed above do not necessarily minimize the Lagrangian, L, we have: $g(x^*) \leq \min L(\mathbf{p}^*, x^*, \mathbf{R})$, for the $p_n(k)$ and the subchannel allocation above. Because of the equalities in (36) and (37), L is simplified to

$$L(\mathbf{p}^*, x^*) = \sum_{n=1}^{N} \alpha_n \sum_{k=1}^{K} p_n(k) - x_i^* \tag{39}$$

On the other hand, $g(x^*) \geq 0$ (because the optimal value of the WSPmin problem is non-negative and $g(x^*)$ is very close to this optimal value. Therefore,

$$x_i^* \leq \sum_{n=1}^{N} \alpha_n \sum_{k=1}^{K} p_n(k) \tag{40}$$

After finding these upper bounds x_n^* for all n, $\overline{\mu}^0$ is chosen by dividing this into two. The matrix P_0 is chosen as $\frac{N}{4}\mathrm{diag}(x_1^{*2}, x_2^{*2}, \ldots, x_N^{*2})$. After finding these initial values, the iterations are performed according to the subgradients found at each step. As in [12] as suitable subgradient is $d_n^\phi = R_n^\phi - \sum_{k=1}^{K} \log_2(1 + p_n^{\phi*}(k)c_n^\phi(k))$. However, we also need to take into account the constraint $\mu_n^\phi \geq 0, \forall n$. At each step we look at if this constraint is satisfied. If not, we choose the subgradient $d_n^\phi = -(-\mu_n^\phi)^+$. If yes, we use the former subgradient. The iterations are performed as explained in [20]

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