Asymptotic Analysis of an OFDMA Scheme with Random Packet Arrivals

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***Abstract*— In this work we perform a queueing analysis of an OFDMA-based and channel-aware resource allocation scheme. We estimate the service characteristics of the queues using extreme value theory and estimate the tail probability of queue size distribution using generating function approach. Based on numerical evaluations it is seen that our estimates are very close to the values obtained by simulations.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technique that is used in current broadband wireless technologies. It is based on a large number of orthogonal subcarriers, each working at a different frequency. OFDM is originally proposed to combat inter symbol interference and multipath fading . However it also has a potential of a multiple access scheme, OFDMA, where the subcarriers are shared among the competing users. Subcarriers are grouped into subchannels in an OFDMA system to simplify the resource allocation process. Subcarriers in an OFDM system experience frequency selective correlated fading. Using adjacent grouping methods (e.g. band-AMC in WiMax) adjacent subcarriers are grouped into a subchannel. Then, it is reasonable to assume that each subchannel experiences flat fading, while different subchannels experience independent and identically distributed (i.i.d.) fading with respect to each user [1]. This property in fact can be exploited to maximize the capacity by allocating each subchannel to the user with best channel gain on that subchannel.

Although this way of resource allocation doesn't guarantee any QoS requirements (e.g. delay or short term received rate), it can still be preferred because of its simplicity. In this work we will perform a queueing analysis of such an OFDMA based system. A fixed power level is used at each subchannel and each subchannel is allocated to the user that maximizes the signal to noise ratio (SNR). Such a system was previously analyzed in [2], where the author studied the asymptotic throughput analysis using extreme value theory [3]. Moreover, for users with different distances to the BS (hence different average SINRs) the author considered allocation of the subchannel to the best *normalized* SINR. Extreme order statistics can be used to approximate the distribution of maximizing random variable in a large set of random variables. Using this method the author in [2] carried out a throughput analysis of the system and proved that asymptotic analysis is quite accurate. In [2] an analysis of delay was also attempted, however apparently it is not realistic. The author models the system as a continuous time M/G/1 system, where each user is of equal distance to the base station. The system is inherently discrete-time, since the channel condition changes and new allocations are made at every time slot. Besides, OFDM is used in wireless metropolitan are networks such as WiMax and LTE, where user distances have a huge variations. In this paper, modeling it as a discrete time multiserver queueing system [4] and using generating function approach we estimate the tail probability of buffer occupancy at a node for channel aware scheduling in OFDMA based systems. Probability of exceeding a certain buffer occupancy threshold is determined as the QoS metric.

The rest of the paper is organized as follows. In Section II we describe our system model. In this section we also describe the extreme value methodology. In Section III we make an analysis for the tail probability of queue size. In Section IV, we evaluate accuracy of tail probability analysis by simulations. We also look at the trade-off between transmission power and supported traffic rate. In Section V we look at the case of heterogeneous average SNRs. If the nodes have different average SNRs (due to differences in distance or log-normal fading) we can revise the scheme to schedule user with best normalized SNR. We numerically compare tail probability estimates with simulations results. This scheme is especially suitable for uplink transmission, since the user can adjust its traffic rate depending on the tail probability estimates.

II. System Model

We consider a system, where N users share a total bandwidth of W Hz, which is divided into K subchannels of bandwidth W_{sub}. A fixed power P per subchannel is used by all nodes. We assume that each subchannel is subject to i.i.d. fading which is constant at each slot of duration T_s and varies from slot to slot. Since fading level is fixed at each slot, we make an AWGN channel assumption and use the tight SNR-BER relations derived in [5]. Let $\gamma_{i,k}$ be the instantaneous SNR of user i at subchannel k. For a target BER the transmission rate in a subchannel as a function of SNR is,

$$r_{i,k} = W_{sub} \log_2(1 + \beta\gamma_{i,k}) \quad (1)$$

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where $\beta = -1.5/\ln(5 \times BER)$. This formulation was proposed for M-QAM modulation however, it also effectively models continuous rate adaptation [6]. The scheduling mechanism is as follows, each subchannel is allocated to the user with maximum SNR on that subchannel. We assume that each user has identical average SNR and identical fading distribution.

We will start from a simple case, the channel condition of each user at each subchannel is i.i.d Rayleigh distributed with mean γ_0 for all i and k, that is $F_\gamma(\gamma_{i,k}) = 1 - e^{-\frac{\gamma_{ik}}{\gamma_0}}$.

A. Extreme Value Theory

In order to analyze such a system we need to derive the probability distribution of the maximizing SNR at each subchannel. We can use extreme value theory in finding the asymptotic distributions of extreme values in a set of i.i.d. variables.

Let's define $\Gamma_k = \max_{i \in \mathcal{N}} \gamma_{i,k}$ as the maximizing SNR in subchannel k, where $\mathcal{N}$ is the set of users. For large $\mathcal{N}$, we can approximate the distribution of Γ_k as an extreme value distribution, if some conditions are satisfied [3]. Let $\gamma_{1,k}, \gamma_{2,k}, \ldots, \gamma_{N,k}$ be independent and identically distributed random variables with distribution function $F_\gamma(x)$. If there exists constants $a_N \in R, b_N > 0$, and some nondegenerate distribution function H such that the distribution of $(\Gamma_k - a_N)/b_N$ converges to H, then H belongs to one of the three standard extreme value distributions:Frechet, Weibull and Gumbel distributions. Since channel conditions are i.i.d. and average SNR's are same for all users we can drop the subchannel subscript. The distribution function of $\gamma_{i,k}$, $F(x)$, determines the exact limiting distribution. If a distribution function $F(x)$ results in one limiting distribution, then $F(x)$ belongs to the *domain of attraction* of this function.

Lemma 1: [3],[2] Let $\gamma_{1,k}, \gamma_{2,k}, \ldots, \gamma_{N,k}$ be i.i.d. random variables with distribution function $F(x)$. Define $\omega(F) = \sup\{x : F(x) < 1\}$. Assume that there is a real number x_1 such that, for all $x_1 < x < \omega(F)$, $f(x) = F'(x)$ and $F''(x)$ exist and $f(x) \neq 0$. If

$$\lim_{x \to \omega(F)} \frac{d}{dx}\left(\frac{1-F(x)}{f(x)}\right) = 0$$

then there exists constants a_N and $b_N > 0$ such that $(\Gamma - a_N)/b_N$ uniformly converges in distribution to a normalized Gumbel random variable as $N \to \infty$. The normalization constants are

$$a_N = F^{-1}\left(1 - \frac{1}{N}\right) \quad (2)$$

$$b_N = F^{-1}\left(1 - \frac{1}{Ne}\right) - F^{-1}\left(1 - \frac{1}{N}\right) \quad (3)$$

where $F^{-1} = \inf\{y : F(y) \geq x\}$

Rayleigh distributed random i.i.d random variables ($f_\gamma(\gamma) = \frac{1}{\gamma_0}e^{-\frac{\gamma}{\gamma_0}}$ and $F_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_0}}$) satisfy the above Lemma.
For $\gamma_{i,k}$ Rayleigh distributed with mean γ_0 , the parameters are: $a_N = \gamma_0 \ln N$ and $b_N = \gamma_0$. Therefore the random variable $\frac{\Gamma - \gamma_0 \ln N}{\gamma_0}$ can be approximated as a normalized Gumbel random variable. A normalized Gumbel distributed random variable, Γ with distribution function $e^{-e^{-\Gamma}}, -\infty < z < \infty$ has expectation $E(\Gamma) = E_0 = 0.5772..$ and variance $\mathrm{Var}(\Gamma) = \frac{\pi^2}{6}$.

Let's redefine $r(\gamma_{i,k}) = \frac{W_{sub}T_s}{L}\log_2(1 + \beta\gamma_{i,k})$ as the number of packets (of length L bits) that can be transmitted by user i in subchannel k. Let's define the rate of the SNR-maximizing user in subchannel k as $R^k_{max,N} = \max_{i \in \mathcal{N}}(r(\gamma_{i,k}))$. Since the SNR's are i.i.d, the distribution of $R^k_{max,N}$ is invariant of subchannels, therefore we can drop the subchannel index k. In [2], it was proven that if the SNR distribution satisfies Lemma 1, then rate of the maximum-SNR user also converges to Gumbel distribution. More specifically $\frac{R_{max,N} - a_N}{b_N}$ converges to normalized Gumbel distribution, where,

$$a_N = \frac{W_{sub}T_s}{L}\log_2(1 + \beta\gamma_0 \ln N) \quad (4)$$

$$b_N = \frac{W_{sub}T_s}{L}\log_2\left(\frac{1 + \beta\gamma_0(1 + \ln N)}{1 + \beta\gamma_0 \ln N}\right) \quad (5)$$

Mean and standard deviation of rate of maximum-SNR user in any subchannel is the following,

$$E\{R_{max,N}\} = b_N E_0 + a_N \quad (6)$$

$$Std\{R_{max,N}\} = b_N \frac{\pi}{\sqrt{6}} \quad (7)$$

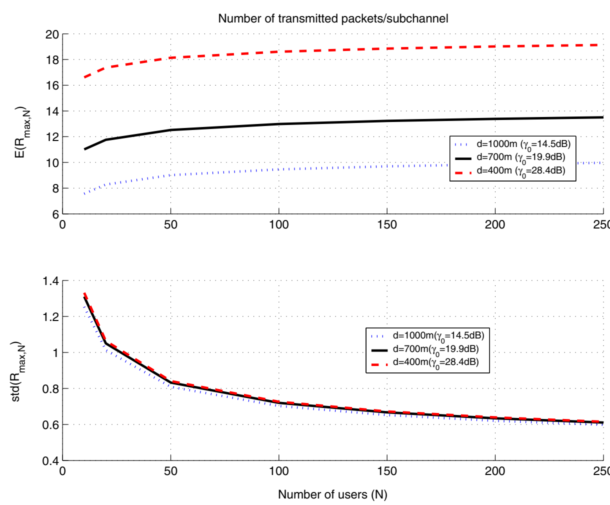

Fig. 1. Mean and standard deviation

Looking at (5), we see that as $N \to \infty$, $a_N \to \infty$ and $b_N \to 0$, and R_N converges to its mean value, $R_{max,N} \approx b_N E_0 + a_N$.

$$E[R_{max,N}] = \frac{W_{sub}T_s}{L}\left(\log_2\left(\frac{1 + \beta\gamma_0(1 + \ln N)}{1 + \beta\gamma_0 \ln N}\right)E_0 + \log_2(1 + \beta\gamma_0 \ln N)\right) \quad (8)$$

Figure 1 shows the mean and standard deviation of $R_{max,N}$. These results numerically verify that standard deviation decreases and mean increases as $N \to \infty$. Standard deviation is smaller than 1 packet even for moderate number of users, therefore we can assume that a user can transmit $\lfloor b_N E_0 + a_N \rfloor - 1$, $\lfloor b_N E_0 + a_N \rfloor$ or $\lceil b_N E_0 + a_N \rceil$ packets in a subchannel, if allocated. Lets define $R(z) = P(R_{max,N} < \lfloor b_N E_0 + a_N \rfloor) z^{\lfloor b_N E_0 + a_N \rfloor - 1} + P(\lfloor b_N E_0 + a_N \rfloor < R_{max,N} < \lceil b_N E_0 + a_N \rceil) z^{\lfloor b_N E_0 + a_N \rfloor} + P(R_{max,N} > \lceil b_N E_0 + a_N \rceil) z^{\lceil b_N E_0 + a_N \rceil}$ as the p.g.f. of number of packets transmitted in a subchannel, if allocated. Each user has equal chance of allocating a subchannel, therefore probability of allocation of channel k by a user is $\frac{1}{N}$ for all users and subchannels. Therefore number of allocated subchannels is Binomial distributed. Let $\sigma(s)$ be the probability of total number of packets that can be transmitted in a time slot being equal to s. Let $\Sigma(z)$ be the probability generating function of $\sigma(z)$.

$$\Sigma(z) = \sum_{k=0}^{K} C(K,k) \left(\frac{1}{N}\right)^k \left(1 - \frac{1}{N}\right)^{K-k} (R(z))^k \qquad (9)$$

III. Queueing Analysis

Since the channel conditions for each user and at every subchannel is i.i.d. and channel allocation is performed purely based on normalized channel condition we can decouple the queues of each user and avoid the problem of interacting queues. In queueing theory this system can be modeled as a multiserver system, where the number of active servers is random according to probability vector σ and an active server can transmit a packet in one time slot. We use the generating function approach that was used in [4] for a different system. Queueing model for our system can be summarized as follows.

1) Arrivals: A random number of L-bit packets arrive at each time slot. The arrivals occur at the end of the time slot, which means that the data unit that arrives in the current slot can be transmitted in the future time slots. Let a_t denote the number of data units arriving at time slot t. Let $A(z) = E[z^a]$ be the probability generating function function (p.g.f.) of a_t, where E[.] denotes the expected value. For poisson distributed arrivals $A(z) = e^{\lambda(z-1)}$, where $E[a] = \lambda$ packets. For geometric distribution it is $A(z) = \frac{1}{1+\lambda-\lambda z}$.
2) Service: We assume that services start at the beginning of a time slot and end before the new arrivals come. Let's define $c = K \times \lceil R_{max,N} \rceil$ as the number of servers and let s_t be the number of packets served at time slot t.

$$s_t = s, \ w.p. \ \sigma(s), s = 0,1,\ldots,\min(q_t, c) \qquad (10)$$

We define the conditional probability generating function $S_i(z)$ (given that there are i packets in the buffer) as,

$$S_i(z) = E[z^{s_t} | \min(q_t, c) = i], \ i = 0,1,\ldots,c \qquad (11)$$

$$= \sum_{s=0}^{i-1} \sigma(s) z^s + \sum_{s=i}^{c} \sigma(s) z^i \qquad (12)$$

Channel allocation is purely based on SNR values and sometimes a user may be allocated more resources than that is enough to empty out the queue. For the simplicity of analysis, in this case we assume that dummy packets are transmitted on the excess subchannels. We also assume that services are independent of arrivals.
3) Overflows: Let D_{max} be the delay constraint in slots. We convert this to a queue size constraint $Q_{max} = \lambda \times D_{max}$ packets using Little's result. Normally, if an arriving packet finds the system full, then it is considered dropped. However, for the simplicity of analysis we are considering an infinite capacity buffer and define the QoS metric as the overflow probability, which is the tail probability of buffer content distribution ($Prob[q_t > Q_{max}]$).

The system equation of the buffer content with respect to time can be written as follows,

$$q_{t+1} = q_t - s_t + a_t \qquad (13)$$

Let $Q_t(z)$ denote the pgf of q_t. Considering the independence of arrival and service processes and using standard z-transform techniques, we can convert the system equation into the z-domain as follows,

$$Q_{t+1}(z) = A(z) E[z^{q_t - s_t}]$$
$$= A(z) \left(Q_t(z) S_c(\frac{1}{z}) + \sum_{i=0}^{c-1} q(i) z^i \left(S_i(\frac{1}{z}) - S_c(\frac{1}{z}) \right) \right), \qquad (14)$$

where $q(i)$ denotes the probability that there are i packets in the queue. We are interested in stable systems, where the buffer content distribution reaches a steady state. When the steady state is reached, $Q_t(z)$ and $Q_{t+1}(z)$ converge to a steady state p.g.f. $Q(z)$. Solving the above equation for equilibrium, we get the expression for Q(z).

$$Q(z) = \frac{z^c A(z) \sum_{i=0}^{c-1} \left(S_i(\frac{1}{z}) - S_c(\frac{1}{z}) \right) q(i) z^i}{z^c - z^c S_c(\frac{1}{z}) A(z)} \qquad (15)$$

$$= \frac{z^c A(z) \sum_{i=0}^{c-1} \left(\sum_{s=i}^{c} \sigma(s) (z^{-i} - z^{-s}) \right) q(i) z^i}{z^c - z^c \sum_{s=0}^{c} \sigma(s) z^{-s} A(z)} \qquad (16)$$

$$= \frac{A(z) \sum_{i=0}^{c-1} \left(\sum_{s=i}^{c} \sigma(s) (z^c - z^{c-s+i}) \right) q(i)}{z^c - \sum_{s=0}^{c} \sigma(s) z^{c-s} A(z)} \qquad (17)$$

where $q(i) = Prob[q_n = i], i = 0,1,\ldots,c-1$ are the buffer occupancy probabilities.

In order to derive $Q(z)$ completely, we need to find the c unknown probabilities $q(i)$ for $i = 0,2,\ldots,c-1$ [4]. Here we need the analyticity property of $Q(z)$ inside the unit disk $(z : |z| < 1)$. A complex function is said to be analytic in a region if it is defined and differentiable at every point in the region. In order to have the analyticity property, poles of Q(z) inside the unit disk must also be the zeros of Q(z). At this point Rouche's theorem [7] stated below can be utilized to show the number of roots of the denominator inside the unit disk.

Theorem 1: Rouche's Theorem[7] says that: If $f(z)$ and $g(z)$ are analytic functions of s inside and on a closed contour C, and also if $|g(z)| < |f(z)|$ on C, then $f(z)$ and $f(z) + g(z)$ have the same number of zeroes inside C. Assuming geometric distributed arrivals $\frac{1}{1+\lambda-\lambda z}$ the denominator of $Q(z)$, $z^c(1+\lambda-\lambda z) - \sum_{s=0}^{c} \sigma(s) z^{c-s}$, has c roots inside and including $(z : |z| <$

1).

Proof: Let's define $f(z) = z^c(1+\lambda)$, $g(z) = -\lambda z^{c+1} - \sum_{s=0}^{c}\sigma(s)z^{c-s}$ and $D(z) = |f(z)| - |g(z)|$. For the value $|z| = 1+\varepsilon$:

$$\begin{aligned} D(z) &= |z^c(1+\lambda)| - |\lambda z^{c+1} + \sum_{s=0}^{c}\sigma(s)z^{c-s}| \\ &\geq |z|^c(1+\lambda) - (\lambda|z|^{c+1} + \sum_{s=0}^{c}\sigma(s)|z|^{c-s}) \\ &\geq (1+\varepsilon)^c(1+\lambda) - (\lambda(1+\varepsilon)^{c+1} + \sum_{s=0}^{c}\sigma(s)(1+\varepsilon)^{c-s}) \\ &= \varepsilon(-\lambda + \sum_{s=0}^{c}\sigma(s)s) + o(\varepsilon) > 0 \qquad (18) \end{aligned}$$

where (18) follows from $(1+\varepsilon)^c = 1 + c\varepsilon + o(\varepsilon)$. We see that under the condition $\sum_{s=0}^{c}\sigma(s)s > \lambda$ (which is also the stability condition) $|f(z)| > |g(z)|$. Since $f(z)$ has c roots, then the denominator has also c zeros. One of them is at $z=1$, and the others are inside the unit disk. Denominator polynomial has order $c+1$, therefore there is a single zero outside unit disk. ■

Let's denote these roots by $z_j, j = 1,2,\ldots,c-1$. Because of the analyticity of $Q(z)$ for $|z| < 1$, the numerator must also be zero at these points.

$$\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(1-z_j^{-s+i})\right)q(i) = 0, \;\; j = 1,2,\ldots,c-1 \qquad (19)$$

We obtain the c^{th} equation from the equality $Q(1) = 1$.

$$\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(s-i)\right)q(i) = \sum_{s=0}^{c}\sigma(s)s - A'(1) \qquad (20)$$

From the stability assumption, the right hand side of (20) has to be greater than zero. From these K equations, the probabilities $q(i),\; i = 0,1,\ldots,K-1$ can be calculated[1].

A. Tail Probabilities of the Queue Size

Let $P(q > Q_{max})$ denote the tail probability of the queue size. Tail probability can be used to approximate the overflow probability of a limited buffer. It has been previously found in [8],[9],[4],[10] that for sufficiently large values of Q_{max}, the tail distribution of queue size can be approximated as,

$$Prob[q > Q_{max}] \approx -R_q \frac{z_q^{-Q_{max}-1}}{z_q - 1}, \qquad (21)$$

where z_q is the real positive pole of $Q(z)$ with the smallest modulus outside the unit disk, i.e. it is the dominant pole of $Q(z)$. R_q is the residue of $Q(z)$ at $z = z_q$. Assuming geometric distributed arrivals the p.g.f of queue size $Q(z)$ has only one pole outside unit circle (therefore it is real), one pole at z=1 and the rest inside the unit circle. It can be derived by evaluating $(z - z_q)Q(z)$ at $z = z_q$.

$$\begin{aligned} R_q &= (z-z_q)Q(z)\big|_{z=z_q} \qquad (22) \\ &= \left.\frac{(z-z_q)A(z)\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(z^c - z^{c-s+i})\right)q(i)}{z^c - \sum_{s=0}^{c}\sigma(s)z^{c-s}A(z)}\right|_{z=z_q} \qquad (23) \\ &= \left.\frac{A(z)\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(z^c - z^{c-s+i})\right)q(i)}{\frac{1}{z}\sum_{s=0}^{c}\sigma(s)sz^{c-s}A(z) - \frac{z^c A'(z)}{A(z)}}\right|_{z=z_q} \qquad (24) \\ &= \frac{A(z_q)\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(1 - z_q^{-s+i})\right)q(i)}{\frac{1}{z_q}\sum_{s=0}^{c}\sigma(s)sz_q^{-s}A(z_q) - \frac{A'(z_q)}{A(z_q)}} \qquad (25) \end{aligned}$$

Equation (24) is obtained by applying the L'Hospital rule and then using the fact that denominator of $Q(z)$ is zero at $z = z_q$. As the system load increases, z_q approaches to 1, the probability of exceeding a buffer occupancy threshold increases. For geometric arrival process (i.e. $A(z) = \frac{1}{1+\lambda-\lambda z}$) the residue is written as follows:

$$R_q = \frac{\sum_{i=0}^{c-1}\left(\sum_{s=i}^{c}\sigma(s)(1 - z_q^{-s+i})\right)q(i)}{\frac{1}{z_q}\sum_{s=0}^{c}\sigma(s)sz_q^{-s} - \lambda} \qquad (26)$$

IV. Numerical Evaluations

We performed a numerical study to evaluate the accuracy of tail probability estimates and see the energy-QoS trade-off by varying the transmission power. We assume a system of K=30 subchannels, where each subchannel is of $W_{sub} = 200KHz$. System is slotted with slot length $T_s = 0.001$sec. Pathloss in (dB's) is $31.5 + 35 * \log_{10}(d)$, where d is the distance of the node to the base station. We assume Rayleigh fading with mean equal to one that is constant at each time slot and is i.i.d. from slot to slot. In Figure 2, we considered 100 users and two packets sizes $L = 100$ and 50 bits,

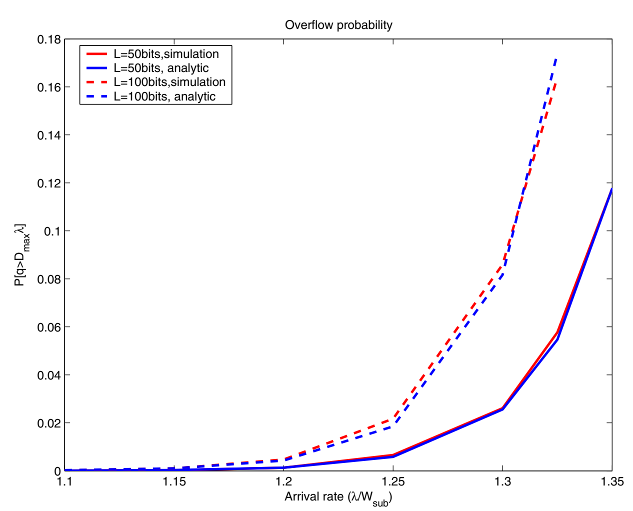

Fig. 2. Tail probability vs. traffic rate

[1]Since we consider a large number of users, allocation probability of a subchannel to a user is very low. Probability of allocation of k subchannels to a user diminishes very quickly as k increases. When solving equations (19), (20) in MATLAB, errors occur because of the precision of the software. To prevent this, we can crop the probability vector σ without losing accuracy. This also speeds up the computation

Distances of users are $d = 1000m$ for each user, therefore their average SNRs are the same. Arrival process for each user is geometric distributed with mean varying from 220Kbps to 260Kbps. Delay constraint is 0.1msec, which is converted to $Q_{max} = \lambda \times 0.1$ bits for each arrival rate. Figure 2 shows the analytical and simulation results for overflow probability versus power per subchannel for this system. We observe that analytical results are very close to the simulation results and overflow probability is increasing and convex as a function of arrival rate.

V. Normalized SNR-based Scheduling

In reality average SNRs of users are different due to differences in distances to the base stations and effects of shadowing. In this case scheduling the best user causes unfairness in the network. However, when we schedule users based on their normalized SNR, resource allocation becomes both fair and analyzable. In this case, subchannel k is allocated to the user $\arg\max_{i \in \mathcal{N}} \frac{\gamma_{i,k}}{\gamma_0^i}$. Since the SNR of a users is the product of normalized SNR and a random variable that is i.i.d. for each user and subchannel, previous results on extreme value statistics and subsequent queueing analyses still holds. If user i is allocated a subchannel, then expected number of packets that it can transmit is $R^i_{max,N}$, which is found by replacing γ_0 by γ_0^i, average SNR of the user that maximizes the normalized SNR.

In this system each user has the same channel access probability, however users with higher average SNR can support sessions with higher rates. The ratio of session rates of users i and j is, $\frac{\lambda_i}{\lambda_j} = \frac{R^i_{max,N}}{R^j_{max,N}}$. If we set the proportionality $\lambda_0^1 : \lambda_0^2 : \ldots : \lambda_0^N = R^1_{max,N} : R^2_{max,N} : \ldots : R^N_{max,N}$ among different user traffic rates, we can better utilize the resources.

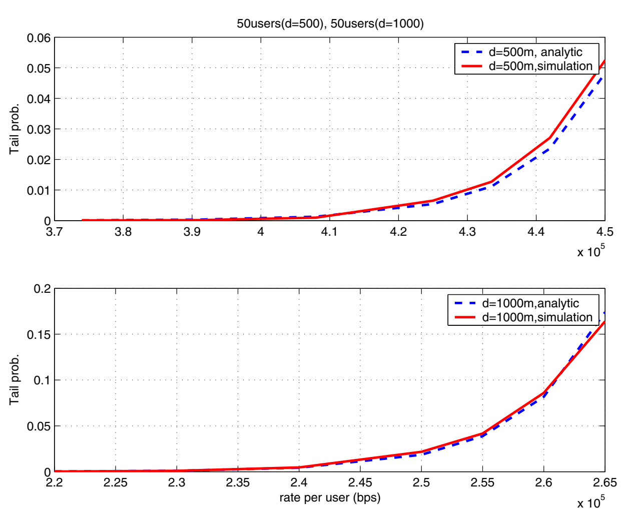

Fig. 3. Tail probability vs. rate for heterogeneous SNR case

In Figure 3 we considered a system of 50 users at 500m and 50 users at 1000m distances. For near users $R^i_{max,N} = 16.6871$ and for far users $R^i_{max,N} = 9.7777$ packets/slot. The ratio is 1.7 and we increase the rate, maintaining this ratio among rates of two classes of users. We see that analytical results closely follow the simulation results.

A. Implementation of the system

A realistic system has to support users with different average SNRs and demanding services with different QoS requirements. For example data services have very loose delay requirements. Besides these sessions can use whatever rate that is available to them. On the other hand video streaming sessions have stricter delay requirements and they can be transmitted in varying quality levels (e.g. 128,256,512,1024Kbps). Since we can estimate maximum supportable rate through $R^i_{max,N}$ for all users, a video user can choose one of the available levels based on this estimate and its QoS requirements. On the other hand voice sessions (e.g. VoIP) have a single rate level, therefore for these sessions overutilization may occur. This problem can be relieved if a voice user doesn't enter the *competition* if it doesn't have any packets in its buffer.

VI. Conclusions

In this work we studied queueing analysis of an OFDMA based resource allocation scheme using extreme value theory and generating function approach. We performed a queueing analysis to estimate the tail probability of queue size distribution for this system. We tested the accuracy of the estimates by simulations and observed that estimates are quite accurate. We both considered systems where users have same average SNR and different average SNRs. The analysis we performed can be used to easily estimate the probability of quality of service violation given the system parameters and to adjust the session rate or transmission power to improve the utilization.

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