This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2009 proceedings.

Fairness and QoS-Based Resource Allocation in Multihop Relay Networks

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***Abstract*—In this paper we study the problem of subframe, subchannel and power allocation in OFDMA-based multihop relay networks. The system consists of a base station (BS), a number of relay stations (RS) and mobile stations (MS). We consider frame by frame scheduling, where the frame is divided into two subframes such as BS-RS and RS-MS subframes. We study two different problems, satisfying link rate requirements with minimum weighted total power and maximizing proportional fairness. For the first problem we find the optimal solution and also propose a less complex subframe and bandwidth allocation scheme with good performance. For the second problem, we propose an algorithm that outperforms an existing scheme with less feedback.**

I. Introduction

Broadband wireless access networks [1] are designed to provide cellular systems that support fixed and mobile users with heterogeneous and high rate traffic requirements. In such networks a single base station (BS) covers a cellular area of radius on the order of miles. Recently, deployment of low-cost relay stations (RS) is considered in order to extend coverage and improve quality of service (QoS) of users that are at the cell edge or that are shadowed by buildings in the suburban areas. Standardization of relay-based broadband wireless access networks is carried out by the 802.16j Relay working group [2], [3].

In IEEE 802.16j-based networks OFDMA is the multiple access and transmission technique. In OFDMA basic resources are subchannels and power. Subchannels experience frequency selective fading, which makes the optimal allocation of these resources crucial in reaching various objectives such as improving throughput, reducing power consumption or maximizing fairness. Multiuser OFDM has been studied in detail for traditional cellular downlink and uplink scenarios before. Previous literature includes studies about subchannel and power allocation for throughput maximization [4], proportional fairness [5], uplink [6] and power minimizations subject to rate constraints in multiuser OFDM systems [8] [7].

Typical relay stations are envisaged to have only one interface, therefore they aren't able to transmit and receive simultaneously. Therefore access (BS-RS, BS-MS) and relay (RS-MS) transmissions are scheduled in a TDMA manner , where these two groups of transmissions occur in different subframes [3]. This adds the dimension of subframe duration to the existing subchannel and power allocation problems. Previous work in the literature mostly did not consider the optimization of subframe duration. The work in [9] develops a subcarrier and power allocation scheme with fixed subframes in order to maximize throughput subject to BS, RS or total power constraints. The authors in [10] and [11] propose heuristics that improve throughput and coverage. The paper [12] analyzes and simulates the uplink capacity of a relay assisted OFDMA system. We have recently proposed a resource allocation scheme that jointly satisfies fairness and rate constraints by allocating subframes, bandwidth and power in [13], however in that work we assumed flat fading, which suggests simpler resource allocation schemes, however doesn't take advantage of frequency selective fading. Minimum power subframe and channel allocation subject to rate constraints was only studied in [15]. In this work the authors propose a subframe, subchannel allocation and bit loading scheme. The authors propose a heuristic to determine the durations of the BS and RS subframes. Then the resource allocation problem becomes separate instants of minimum-power downlink allocation problems, which is studied in [7].

In this work, by extending [8] we propose a polynomial-time algorithm that jointly optimizes subframe time, subchannel and power allocations. The objective is to minimize total weighted transmission power subject to link rate constraints. We develop a suboptimal but less complex joint subframe and bandwidth allocation scheme that has better weighted total power expenditure performance than in [15].

In the second part of the paper we study resource allocation for fairness in relay networks. The authors in [14] proposed a proportional fair resource allocation scheme before. Centralized resource allocation schemes for cellular relaying systems require feedback from the RSs, which becomes complex when channel conditions for each user and subchannel has to be fed back. In this work we propose a fair allocation scheme that achieves better performance than in [14] with less feedback.

The paper is organized as follows. In Section II we present the system and frame model. In Section III we optimally solve the problem of weighted sum power minimization subject to rate constraints. In Section IV we propose a suboptimal way of allocating subframes and bandwidth which is followed by numerical evaluations. In Section VI we present the problem of fair resource allocation in the same setting.

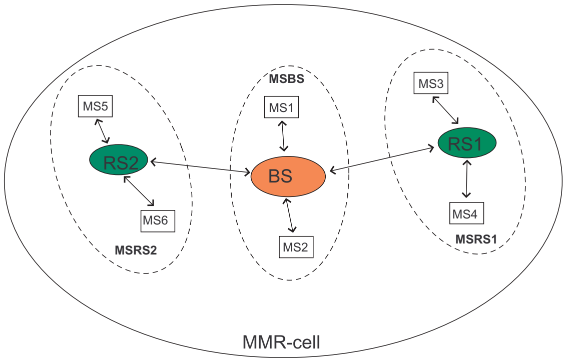

Fig. 1. Topology of a MMR cell. The BS is serving the MSs in the set MS_{BS} directly (MS_1 and MS_2). Two relay stations (RS_1, RS_2) are used to enhance the system throughput of BS and serve MSs in the set MS_{RS1} (MS_3, MS_4) and MS_{RS2} (MS_5, MS_6). The MMR cell includes the coverage area of the BS and all the RSs.

II. System Model

We consider a mobile multihop relay (MMR) system consisting of a BS and M RSs that are fixed. There are N user stations (MSs), each of which is assigned to either the BS or one of the RSs according to the distance. A sample MMR cell is shown in Figure 1. We consider frame by frame resource allocation. A frame is of duration T_f and it is divided into two subframes as in Figure 2. In the first subframe the transmissions from the BS occur (BS-RS and BS-MS). In the second subframe the transmissions from the RSs occur. We consider an OFDMA system where a total bandwidth of W is divided into K subchannels. The transmissions of all RSs occur simultaneously but in disjoint subchannels. We assume that the transmissions experience path loss, Rayleigh fading and log-normal shadowing. Channel conditions are assumed to be constant during a frame. Rayleigh fading is assumed to be flat in each subchannel and i.i.d for different users and subchannels. For the system and transmission quantities like channel condition, power and rate we define a superscript ϕ that becomes BS or RS depending on which link is intended (access or relay). Let the spectral efficiency achieved by user n at subchannel k be $S_n^{\phi}(k) = \log_2\left(1 + p_n^{\phi}(k)c_n^{\phi}(k)\right)$, where $\phi = BS, RS$. Here $p_n^{\phi}(k)$ is the transmission power allocated to user n at subchannel k and link ϕ. Parameter $c_n^{\phi}(k)$ is the channel condition $c_n^{\phi}(k) = \frac{\beta h_n^{\phi}(k)}{N_0 W/K}$, where $h_n^{\phi}(k)$ is the combined channel gain and $N_0 W/K$ is the noise power at a subchannel of bandwidth W/K Hz. Parameter β is the SNR-gap which is taken as 0.25, based on the predefined modulation-coding pairs and corresponding SNR thresholds in IEEE 802.16 standard. We assume that the BS can perfectly obtain the channel condition parameters of all RSs and MSs.

III. Sum Power Minimization

Let R_n^{ϕ} be the target rate of user n at link ϕ in bits per subchannel. The aim is to satisfy these rate requirements by using minimum total weighted power. The choice of weights is important ; for example in the downlink, the weights of the BS-RS, BS-MS and MS-RS transmissions (α_n^{BS} and α_n^{RS} resp.) can be chosen to adjust power expenditure by the BS and RS. Since the RSs are deployed closer to the cell edge their power must be adjusted to limit interference to other cells.

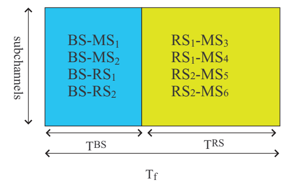

Fig. 2. An MMR frame is divided into two subframes, which are called BS and RS subframes. A link transmits in using certain set of subchannels throughout the subframe duration.

Resource allocation constraints are frame duration ($T^{RS} + T^{BS} \leq T_f$) and the constraint that each subchannel should be used by at most one transmission. Optimally dividing the frame is important because a linear change in transmission time results in an exponential change in power expenditure for a target rate. Power minimization problem becomes:

$$\min_{T^{RS},\mathbf{p^{BS}},\mathbf{p^{RS}}} \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^{BS} p_n^{BS}(k) + \alpha_n^{RS} p_n^{RS}(k) \quad (1)$$

$$T^{\phi}\sum_{k=1}^{N} \log_2\left(1 + p_n^{\phi}(n)c_n^{\phi}(k)\right) \geq R_n^{\phi},\ \phi = BS, RS, n = 1,\dots,N \quad (2)$$

$$\mathbf{p}^{\phi} \in \mathcal{D},\ \phi = BS, RS \quad (3)$$

where $T^{BS} = T_f - T^{RS}$. Here $\mathcal{D}$ is the set of power allocations such that for all $k = 1,\dots,K$ and $\phi = BS, RS$, $p_n^{\phi}(k) > 0$ for only one user in $n = 1,\dots,N$ [8]. We solve this problem by using Lagrange dual decomposition method. We can write the Lagrange dual as follows:

$$L(\mathbf{p},\bar{\mu},T^{RS}) = \sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^{BS} p_n^{BS}(k) + \alpha_n^{RS} p_n^{RS}(k) - \sum_{\phi=BS,RS}\sum_{n=1}^{N} \mu_n^{\phi}\left(T^{\phi}\sum_{k=1}^{K}\log_2\left(1 + p_n^{\phi}(k)c_n^{\phi}(k)\right) - R_n^{\phi}\right) \quad (4)$$

where $\bar{\mu} = \mu_n^{\phi}$, $\phi = BS, RS$ $n = 1,\dots,N$. For a given relay subframe time T^{RS}, the problem reduces to minimum power allocation in [8]. By taking derivative with respect to $p_n^{\phi}(k)$, we obtain an expression for transmission power. By plugging this expression in (4), we obtain the Lagrangian in terms of Lagrange multipliers $\mu_n(k), \forall n, k$ and $T^{\phi}, \phi = BS, RS$.

$$L(\mathbf{p},\mu,T^{RS}) = -\sum_{\phi=BS,RS}\sum_{n=1}^{N} \mu_n^{\phi} T^{\phi} \sum_{k=1}^{K}\left(\log_2\left(\frac{\mu_n^{\phi} c_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}}\right)\right)^{+} + \sum_{\phi=BS,RS}\sum_{n=1}^{N}\sum_{k=1}^{K} \alpha_n^{\phi}\left(\frac{\mu_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}} - \frac{1}{c_n^{\phi}(k)}\right)^{+} + \sum_{\phi=RS,BS}\sum_{n=1}^{N} \mu_n^{\phi} R_n^{\phi}$$

For a given T^{RS}, minimization of the Lagrange dual above can be decomposed into independent problems for each subcarrier and subframe. Each subcarrier in a subframe has to be occupied by at most one user. The optimal user that occupies subcarrier k given T^{RS} Lagrange multipliers $\overline{\mu}$ is denoted by $A_k(\overline{\mu}, T^{RS})$ and found as [8],

$$A_k^{\phi}(\overline{\mu}, T^{RS}) = \arg\min_n \left\{ \alpha_n^{\phi} \left(\frac{\mu_n^{\phi} T^{\phi}}{\ln 2\alpha_n^{\phi}} - \frac{1}{c_n^{\phi}(k)} \right)^+ - \mu_n^{\phi} T^{\phi} \left(\log_2 \left(\frac{\mu_n^{\phi} T^{\phi} c_n^{\phi}}{\ln 2\alpha_n^{\phi}} \right) \right)^+ \right\}, \quad (5)$$

for $k = 1, \ldots, K, \phi = BS, RS$. At each subcarrier the minimizing user in (5) is the transmitting user. For a given T^{RS} value we find the optimal vector $\mu_n^{\phi}, \phi = BS, RS; n = 1, \ldots, N$ using the Ellipsoid method [16]. We refer to this scheme as **Optimal**. The details of implementation are omitted due to space constraints.

Proposition 1: Optimal T^{RS} is obtained if the following equality is satisfied:

$$\sum_{n=1} \frac{\mu_n^{BS} R_n^{BS}}{T_f - T^{RS*}} = \sum_{n=1} \frac{\mu_n^{RS} R_n^{RS}}{T^{RS*}} \quad (6)$$

The optimal T^{RS*} is unique and can be found using bisection on $[0, T_f]$.

Proof: Omitted due to space constraints. ■

Subframe length must be an integer number of time slot length. In order to satisfy this requirement, we need to round the subframe length and then perform another round of optimization for subchannels and powers. Another solution to find the optimal subframe length is exhaustive search in $[1, \frac{T_f}{T_s} - 1]$, where T_s is the time slot length. This has a complexity of $O(T_f/T_s)$, while binary search has complexity $O(\log_2 T_f/T_s)$. So our solution decreases the complexity significantly as the number of slots per frame increases.

IV. Suboptimal Resource Allocation

Finding the optimal subframe allocation requires application of ellipsoid method at each search step. Ellipsoid method is a robust and successful method, however it takes $O(N^2)$ iterations to converge , where N is the number of users. Therefore it can be prohibitively complex and a much less complex, yet successful suboptimal solution is needed. In order to reduce the computational complexity, we propose a suboptimal subframe allocation algorithm, where we replace subchannel channel conditions $c_n^{\phi}(k)$ with an average user channel condition $\overline{c}_n^{\phi} = \frac{1}{K}\sum_k c_n^{\phi}(k)$ for all users. Averaging produces a channel with *flat* fading. Then it is possible to simplify resource allocation by assuming bandwidth as a continuous quantity [13] and converting the problem from discrete subchannel allocation to continuous bandwidth allocation. Following joint subframe, power and bandwidth allocation problem can be formulated:

$$\max \sum_{\phi=BS,RS} \sum_{n=1}^{N} -\alpha_n^{\phi} p_n^{\phi} \quad (7)$$

$$T^{\phi} w_n^{\phi} \log_2 \left(1 + \frac{p_n^{\phi} \overline{c}_n^{\phi}}{w_n^{\phi}} \right) \geq R_n^{\phi}, \ \phi = BS, RS, \ n = 1, .., N \quad (8)$$

$$\sum_{n=1}^{N} w_n^{\phi} \leq K \quad (9)$$

$$T^{RS} + T^{BS} \leq T \quad (10)$$

This problem has a concave objective function with convex constraint set. The Lagrange dual is,

$$L(\mathbf{p}, \mathbf{w}, \overline{\lambda_w}, \overline{\lambda}) = \sum_{\phi=BS,RS} \left(\sum_{n=1}^{N} -\alpha_n^{\phi} p_n^{\phi} + \lambda_w^{\phi} \left(K - \sum_{n=1}^{N} w_n^{\phi} \right) \right) + \lambda_T (T - T^{RS} - T^{BS}) + \sum_{\phi=BS,RS} \sum_{n=1}^{N} \lambda_n^{\phi} \left(T^{\phi} w_n^{\phi} \log_2 \left(1 + \frac{p_n^{\phi} \overline{c}_n^{\phi}}{w_n^{\phi}} \right) - R_n \right) \quad (11)$$

Using the equations $\frac{\partial L}{\partial p_n^{\phi}} = 0$, $\frac{\partial L}{\partial w_n^{\phi}} = 0$ and $\frac{\partial L}{\partial T^{\phi}} = 0$ we obtain two relations between T^{ϕ} and (λ^w, λ_T) for $\phi = BS, RS$. Using $T_f = T^{BS} + T^{RS}$ we obtain the optimal power, bandwidth and subframe allocations by using bisection on λ_T. The details can be found in [17]. We will refer to this algorithm as Joint Subframe, Bandwidth and Power Allocation (**JSBP**).

After finding the subframe times, the problem reduces to two separate multiuser OFDMA power minimization problems, which can be solved using [8]. In order to further simplify the problem, successful heuristics proposed in [7] can be used to further simplify the solution.

A. Subchannel and Power Allocation

The JSBP scheme proposed in the previous section produces bandwidth allocations ($\mathbf{w}^{BS}, \mathbf{w}^{RS}$) and powers ($\mathbf{p}^{BS}, \mathbf{p}^{RS}$). Bandwidths that we found are not necessarily integer multiples of subchannel bandwidths. These bandwidth values can be quantized in order to find the number of subchannels allocated to each user. After quantization if $\sum_{n=1}^{N} w_n^{\phi} < K$, then at each step find the user maximizing $w_n^{\phi}(2^{R_n^{\phi}/w_n^{\phi}T^{\phi}} - 1)/\overline{c}_n^{\phi}(k)$(power) and allocate one more subchannel until equality is satisfied. If $\sum_{n=1}^{N} w_n^{\phi} > K$ then at each step find the user with $w_n^{\phi} > 1$ and maximizing w_n^{ϕ} , and decrease its subchannels by one. We then recalculate the power requirement to satisfy those rates for each user as $p_n^{\phi'}$. As in [7] we have the number of subchannels and number of bits per symbol for each user. Hence the heuristic methods proposed in [7] can be used. One of the methods mentioned in this paper is Vogel's method.

Because of space limitations we will only briefly explain Vogel's method. Here first power costs for each subchannel-user pair is defined. Based on these costs, penalties are defined for each user. This is the penalty of not allocating any subchannel to a node in a certain number of steps. At each step the user with highest penalty is chosen and that user gets the available subchannel with minimum cost. Penalties are recalculated at each step. We will refer to this scheme as **Vogel**. After the subchannel allocations the power values are optimized for each user, using water-filling.

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V. Numerical Results

In this section we test and compare the performance of the optimal and suboptimal resource allocation algorithms. We consider the following algorithms,

1) Subframe Allocation: 1) **JSBP**: This is the joint sub-channel bandwidth power allocation scheme we proposed in Section IV. 2) **Mueller**[15] : This subframe allocation scheme first determines a fixed number of bits per Hz for all links. Then it tries all possible number of slots for subframe durations and chooses the one that requires minimum power. 3) $\mathbf{T^{BS}} = \mathbf{T^{RS}}$: This scheme simply divides the frame into two and will be used as a benchmark.

2) Subchannel and Power Allocation:: 1) **Vogel** [7]: This was explained in Section IV-A. If JSBP is used than number of subchannels and power values that are necessary for the Vogel's algorithm are already found. If Mueller's algorithm is used, then these values can be found by plugging the subframe durations in problem (7)-(9) and solving the problem using similar convex programming techniques. 2) **VogelDist**: This is our extension for Vogel's algorithm. Once the number of subchannels are computed, we know the total number of subchannels required for users belonging to each relay station. Then, each relay can perform its own allocations from its given set of subchannels. This limits the subchannel diversity, however it has the advantages of a distributed scheme; the RSs don't need to feedback channel conditions of each user and subchannel to the BS. They only need to feedback average values ($\bar{c}_n^{RS}$). Please note that allocations for the BS subframe is the same.

We perform simulations based on the parameters in Table I. We consider a tandem topology with cell radius of 2000m with BS at the origin. Two relay stations are located at (-1400m, 0m) and (1400m, 0m) points.

Parameter	Value
Cell radius	2km
User Distances	0.4,0.8,1.2,1.6,2.0km
RS Distance	1.4km
# Relay stations (M)	2
W, K	1MHz, 128
Frame Length T_f	4 msec
Slot Length T_s	0.1 msec
AWGN p.s.d.(N_0)	-174dBm/Hz
BS-RS PL(d)(in dB)	$36.5 + 23.5\log_{10} d + \psi_{dB}^{BS-RS}$
RS-MS PL(d)(in dB)	$31.5 + 35\log_{10} d + \psi_{dB}^{RS-MS}$
BS-MS PL(d)(in dB)	$31.5 + 35\log_{10} d + \psi_{dB}^{BS-MS}$
ψ_{dB}^{BS-MS}, ψ_{dB}^{RS-MS}	$\sim N(0dB, 8dB)$
ψ_{dB}^{BS-RS}	$\sim N(0dB, 3.1dB)$
SNR gap coefficient (β)	0.25

TABLE I
Simulation Parameters

Figure 3 show the distributions of weighted total powers for the case of 4 users and and two relay stations. Users are at locations $\mp$2000 and $\mp$400m where 2 of them are assigned to RSs and 2 of them to the BS. All users (and for each user BS-RS and RS-MS links) have a total rate constraint of 3Mbps. For user assigned to relays stations $R_n^{RS} = R_n^{BS}$. We can observe from Figure 3 that Optimal is the best, as expected. We also observe that the performances of Muller and JSBP are quite close to that of Optimal. In the second subgraph we take a close look at the performances. We see that for this case JSBP is better than Muller. $T^{RS} = T^{BS}$ results in the worst performance.

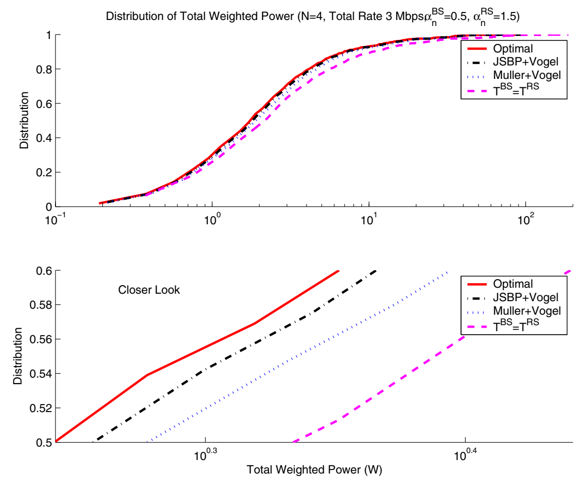

Fig. 3. Total weighted power for three different algorithms (4 users and 2 relay stations).

Table II shows the mean power expenditures for the BS and both stations for the same set of parameters. We see that using JSBP and Vogel results in almost optimal performance however, using $T^{RS} = T^{BS}$ results in fifty percent more energy expenditure.

	Optimal	JSBP	Muller	$T^{BS} = T^{RS}$
mean(P^{BS})	0.78794	0.66583	0.52345	0.53426
mean(P^{RS})	2.7581	3.0324	3.3662	4.8611

TABLE II
Mean BS and RS weighted total power for N=4

In Figure 4 we observe the relation of total weighted power expenditure and number of users. For this purpose, we considered a total system load of 6Mbps, and increased the number of users from 10 to 50. Individual user rates are generated such that they are uniformly distributed and also their sum is 6Mbps. User locations are still at the discrete levels mentioned above and the ratio of users assigned to each station keeps same. As number of users increase the average rate per user decreases. We observe that JSBP scheme achieves 20 percent less power than Muller's scheme at high number of users. As N increases from 10-30, the multiuser diversity is in effect, hence power expenditure decreases. However as N increases from 30 to 50, most users begin to have only one or two subchannels, which makes the subchannel allocation more critical. If each user had high number of subchannels then water-filling would mitigate the effects of subchannel allocation. When each user has one or two channels then waterfilling doesn't have much or any effect.

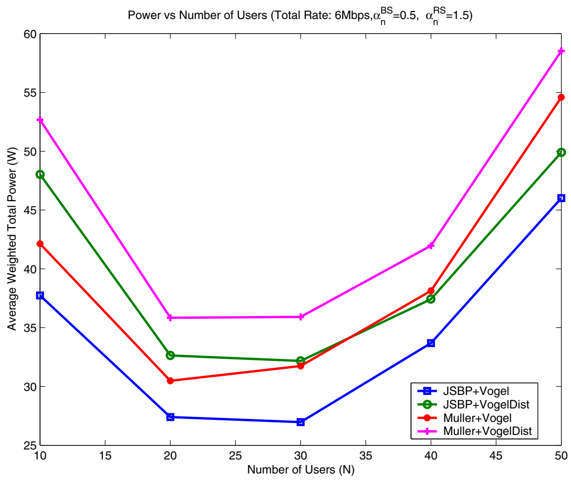

Fig. 4. Total weighted power vs. Number of Users

VI. Fair Subchannel Allocation

In this section we consider the problem of fair resource allocation for delay tolerant data traffic. We study the problem of subframe and subchannel allocation for constant power per subchannel. We redefine $c_n^\phi(k)$ including this power as $c_n^\phi(k) = \frac{\beta h_n^\phi P^\phi}{N_0 W}, \phi = BS, RS; n = 1, \ldots, N; k = 1, \ldots, K$. There are a few papers in the literature that studies fair resource allocation in relay enhanced networks and [14] is one of them. We will briefly explain the proposed algorithm in [14] and propose an improvement for it. In [14] slot is divided *equally* into two subslots. In the first subslot the transmissions $BS \to MS_{BS}$ and $BS \to RS$ occur. In the second one the transmissions $RS \to MS_{RS}$ and $BS \to MS_{BS}$ occur. Hence, $BS \to MS_{BS}$ transmissions can occur in both time slots. First, the allocations in the second subslot occur, which is performed channel by channel. For channel k each user has the following metric

$$\eta_{n,k} = \begin{cases} \frac{W_{sub} \log_2\left(1+c_n^{BS}(k)\right)}{\alpha \bar{R}_n + (1-\alpha) r_n} & n \in MS_{BS} \\ \frac{W_{sub} \log_2\left(1+c_n^{RS}(k)\right)}{\alpha \bar{R}_n + (1-\alpha) r_n} & n \notin MS_{BS} \end{cases} \quad (12)$$

$\bar{R}_n$ is the time averaged rate received by user n, and r_n is the rate allocated to user n in the current slot and it is updated when each subchannel is allocated to user n. Channels in the second subslot are allocated in a greedy manner based on (12).

The relay station that we consider (as well as in [14]) is a "prompt" relay, which means that the information transmitted from BS to RS is immediately decoded and retransmitted from RS to respective MSs. The effective data rate is the minimum of the BS-RS and RS-MS rates. Therefore in the first subslot sufficient resources should be allocated for each RS. The rest of the resources are shared among $BS \to MS_{BS}$ transmissions again according to metric (12).

This algorithm can be improved in a number of ways. 1) Current frame format of IEEE 802.16j is divided into BS and RS subframes instead of dividing into slots. 2) BS and RS only transmit in their own subframes as in our frame format in Figure 2. 3) Performance can be improved by adjusting the subframe durations T^{RS} and T^{BS}. 4) Algorithm in [14] requires feeding back the channel conditions of all users at all subchannels to the BS, which can be excessive. RSs have to feed back $N \times K$ quantities in total. Considering these issues we develop a method that determines the subframe times and amount of bandwidth allocated to each RS. As in the previous power minimization problem we average out the channel conditions of each user over all the subchannels and define $S_n^\phi = \log_2\left(1+\bar{c}_n^\phi\right), \phi = BS, RS$. We assume bandwidth as a continuously divisible quantity. Then we formulate the following proportional fair bandwidth allocation problem. Since the power is uniformly distributed time-frequency product $T^\phi w_n^\phi$ can be defined as a "resource" b_n^ϕ.

$$\max_{\mathbf{b}} \sum_n \log\left(\alpha R_n + (1-\alpha) r_n\right) \quad (13)$$

$$b_n^{BS} S_n^{BS} \geq r_n, \ \forall n \quad (14)$$

$$b_n^{RS} S_N^{RS} \geq r_n, \ \forall n \notin MS_{BS} \quad (15)$$

$$\sum_{\forall n} b_n^{BS} + \sum_{n \notin MS_{BS}} b_n^{RS} \leq W T_f \quad (16)$$

Solving this convex optimization problem using standard methods we obtain the following relations,

$$b_n^{RS}(\lambda_b) = \left[\frac{1}{\lambda_b\left(1+\frac{S_n^{RS}}{S_n^{BS}}\right)} - \frac{\tilde{\alpha} R_n}{S_n^{RS}}\right]^+, \ \forall \notin MS_{BS} \quad (17)$$

$$b_n^{BS}(\lambda_b) = \left[\frac{1}{\lambda_b\left(1+\frac{S_n^{BS}}{S_n^{RS}}\right)} - \frac{\tilde{\alpha} R_n}{S_n^{BS}}\right]^+, \ \forall \notin MS_{BS} \quad (18)$$

For $n \in MS_{BS}$ we find $b_n^{BS}(\lambda_b) = \left[\frac{1}{\lambda_b} - \frac{\tilde{\alpha} R_n}{S_n^{BS}}\right]^+$. We find the optimal λ_b^* such that (16) is satisfied. Then we find the subframe durations as $T^{BS} = \frac{1}{W}\sum_{\forall n} b_n^{BS}$ and $T^{RS} = \frac{1}{W}\sum_{n \notin MS_{BS}} b_n^{RS}$. We need to quantize these values to integer multiples of time slots. Then the bandwidth allocated to each RS is calculated as $W_m^{RS} = W\left(\sum_{n \in MS_{RS_m}} b_n^{RS}\right) / \left(\sum_{n \notin MS_{BS}} b_n^{RS}\right), \ \forall m = 1, \ldots, M$. These bandwidth values also need to be quantized to integer multiples of subchannel bandwidth. Then each RS is given that many subchannels and allocates these subchannels itself to its users according to the metric in (12). Users $n \in MS_{BS}$ are not served in the RS subframe. Then in the BS subframe BS first allocates enough subchannels for the RSs to support the rates allocated to RS users. The remaining subchannels are allocated to users MS_{BS} according to metric (12). Let's denote this algorithm as **Algorithm A**. This algorithm is semi-distributed since an RS can make its own allocations once it is given a set of subchannels. The RSs have to only feed back N quantities for the average user channel conditions ($\bar{c}_n^{RS}, n \in MS_{RS1}$) and N quantities for the allocated rates to users (r_n^{RS}) so that BS can allocate enough subchannels to each RS and transmit enough information for each user in those subchannels. As a benchmark, we propose another centralized algorithm, wherein all allocation is again performed by the BS according to the metric (12) but the subframe durations are still calculated using our proposed method. We call this **Algorithm B**. In Figure 5 we simulated and compared the three algorithms in terms of the total average throughput vs. number of users. We make a saturated queue assumption in order to concentrate on the achievable proportional fair throughput. We performed the simulations for $W = 1MHz$, $K = 128$ and $\alpha = 0.99$ as in [14]. In the BS subframe 20W power and in RS subframe 10 W power is uniformly distributed to each subchannel. We see that despite Algorithm B requires less feedback than [14] it can provide significant improvement. We also see that Algorithm B achieves the best performance, which is expected because BS can use the complete set of subchannels, while in Algorithm A each RS can only use its allocated set of subchannels.

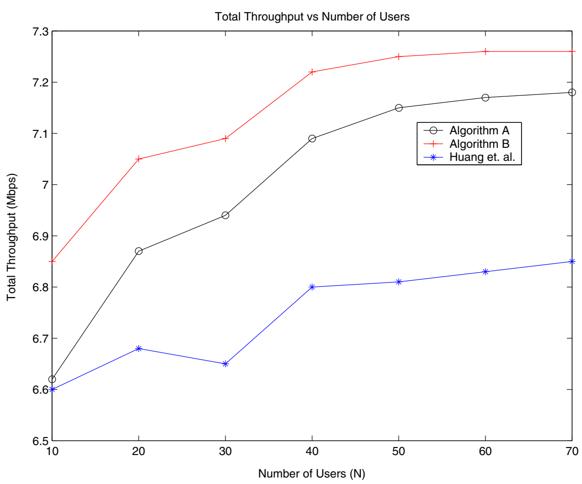

Fig. 5. Total throughput vs. number of users for the three algorithms).

Table III compares the sum of logarithms of average user throughputs, which is a measure of proportional fairness. We again see that Algorithms A and B have better fairness performance than [14]. We understand that the proposed algorithms achieve better throughput without sacrificing fairness.

	$N = 20$	$N = 40$	$N = 60$
Algorithm A	129.3	231.8	324.1
Algorithm B	129.8	232.5	324.9
[14]	128.9	231.0	321.7

TABLE III
SUM OF LOGARITHMS OF AVERAGE USER THROUGHPUTS VS. N

VII. CONCLUSIONS

In this paper we studied OFDMA based resource allocation in cellular systems with relays. Resource allocation involves optimally dividing the frame into BS and RS subframes, allocating subchannels to individual transmissions and loading each subchannel with optimum power. We considered two problems, which are minimizing power subject to rate constraints and maximizing fairness subject to power constraints. In the first problem we found the optimal solution and also proposed a suboptimal solution that performs close to the optimal. For the second problem we considered a recently proposed algorithm as a benchmark and proposed a subframe and subcarrier allocation algorithm that outperforms it. The results of this research is applicable to mobile multihop relay (MMR) networks that are recently being standardized as IEEE 802.16j. Future work should include networks with heterogeneous traffic requirements, which requires joint solution of these two problems. Exploiting cooperative diversity is also made possible in the IEEE standard and is a future work.

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