Minimum-Outage Unicast Routing in Cooperative Wireless Networks

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***Abstract*—We consider the problem of cooperative routing with minimum end-to-end outage in a wireless network. Only the average channel condition is available to the transmitters, therefore there is a possibility of outage on every link from source to destination. Nodes have fixed transmission power so the problem is to determine the order of nodes to transmit from the source to the destination. Each node along the way can hear and combine the two transmitters that transmit just before it. We find the optimal solution based on the Branch-Bound method and also consider some suboptimal algorithms. The simulation results show that one of the suboptimal algorithms can perform very closely to the optimal one.**

Index Terms: Wireless networks, cooperative, multihop, routing, outage

I. Introduction

Wireless channel presents a severe pathloss and fading environment that needs to be coped with. First of all in wireless networks required power to transmit data is proportional to d^{α}, where d is the distance and α is the path loss exponent ($2 < \alpha < 4$). The authors in [1] and in several other papers proposed routing algorithms that route packets through multiple shorter hops in order to save energy. Another issue in wireless networks is the *node based* nature, where the notion of *link* depends on the transmission power and a transmission is heard by many nodes in the environment. Although this creates interference, it also can be exploited in multicast transmissions. This is called as *"wireless broadcast advantage"* and introduced in [2]. The third important issue (in multihop transmission) is that the receivers can combine the transmissions in the previous hops and obtain a higher-quality signal. This is called *cooperative diversity*, which got a lot of attention in the recent years. The authors in [3], proposed a cooperative energy- efficient multihop broadcasting method using wireless cooperative advantage. This method assumes that the nodes in the network can accumulate energy from previous-hop transmissions and if the collected energy is over a required threshold, then decoding is performed reliably. Also [4] investigates cooperative routing, where nodes dynamically form local coalitions and cooperatively transmit packets to the next hop destination. Authors used standard Dijkstra shortest path algorithm for finding energy efficient routes. In [5] a routing algorithm is proposed, which finds minimum power routes using simultaneous transmissions in a static wireless network.

Up to now we mentioned about works which assumed all channel conditions are known and fixed. In fast fading environment where channel states change from one transmission to another, it is very hard to know channel states all the time. Due to the fast fading, SNR (Signal to Noise Ratio) can fall below the required threshold, which is named *outage*. Outage notion becomes more important for real life applications since wireless links are subject to multipath fading. Outage is the dominant cause of errors in wireless communications.

Authors of [6] studied communication reliability of wireless networks and showed end-to-end outage probability is a good metric to characterize reliability of routing algorithms. They also developed algorithms for finding the most reliable (but noncooperative) route subject to power constraints and reliability constraints. In [7] a distributed cooperative routing algorithm is proposed. This algorithm selects the best relays with minimum power consumption in distributed manner and forms cooperative links for establishing a route with a target BER (Bit Error Ratio) from a source to a destination node. In [8] a framework developed in which the transmission of data from the source to destination occurs in a series of hops , where each of the hops use local cooperations between nodes to achieve a global optimization of the cooperative route. Authors in [10] used a different approach to find an optimal solution for the problem. They proposed an algorithm named the Minimum Power Cooperative Routing (MPCR). MPCR uses combination of direct transmission and cooperative transmission blocks to minimize total transmission power while satisfying local outage requirements. Direct transmission and cooperative transmission blocks consist of point-to-point transmissions, cooperative transmission with help of a relay node respectively. They considered the route as a cascade of these blocks, and the total power of the route is summation of transmission powers along the route. They used a distributed shortest path algorithm to solve this minimization problem.

Our study has different aim from others'. In this study our main aim is minimizing the end-to-end outage probability assuming the nodes transmitting with fixed power in our algorithms. We studied multihop unicast routing strategies in a fast fading environment and proposed an implementation of Branch-and-Bound method [11], that we used before in [12], as an *optimal* solution. We proposed several suboptimal algorithms and compared the performances with the optimal one.

The rest of the paper is organized as follows. In the next section we describe the network model and formulate the minimum-outage routing problem. In Section III, we propose cooperation based algorithms and in Section IV we derive the analytical results for the outage performance of the proposed algorithms. Finally, Section V concludes the paper.

II. System Model

System model consists of randomly located nodes on a circular area. In Figure 1, a sample wireless network topology and three different routes are illustrated. The leftmost node and the rightmost node are determined as the source and the destination nodes, respectively. The thick solid lines represent the actual transmissions and the dotted lines represent the transmissions that are overheard from the two previous transmitter. In this example (and in our study) a node can overhear and combine two previous transmission, but this number can be increased in order to improve reliability.

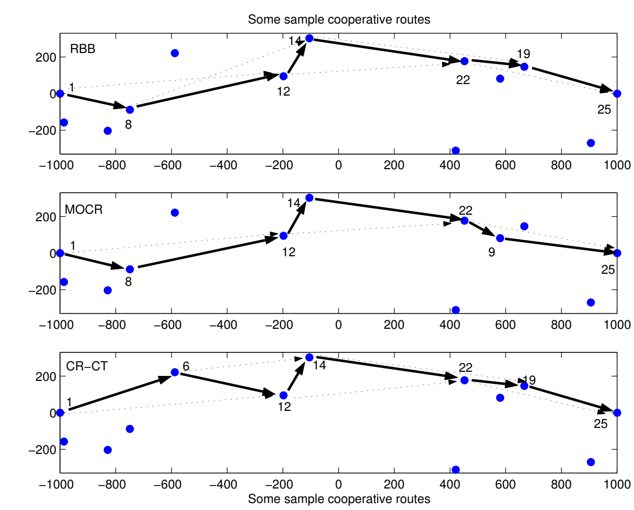

Fig. 1. Illustration of wireless network topology and some cooperative routes. The three routes slightly differ and the first one has the least outage.

We consider a multihop scenario, where the nodes between 1 and N can be relay nodes. Each relay along the route can receive multiple copies of the same data therefore significant improvement of reliability is achieved. The channel model includes distance attenuation and Rayleigh fading. We will both consider some algorithms that can only be implemented in a centralized way and some that can have distributed implementation. The node locations (hence pathloss values) are fixed and they are assumed to be known by the decision maker(s). Let $g_{i,j}$ and $h_{i,j}$ be the distance-based attenuation and Rayleigh fading between nodes i and j. To provide simplicity for our calculations, we don't consider the issue of interference , collision and multiple access.

For the achievable rate of transmission from nodes i and j to node n, we use the Shannon model , where the rate is equal to $W\log_2(1+\frac{g_{i,n}h_{i,n}+g_{j,n}h_{j,n}}{N_oW})$ bps. For the cooperative transmissions achievable rate depends on the sum of received SNRs from the transmitters which transmitted before the current node. Accumulated SNR at node i can be written as

$$\Gamma_i = \frac{P}{N_oW} \times \sum_{n=1}^{|\mathcal{O}_i|} g_{n,i}h_{n,i} \tag{1}$$

where P, N_o and W are the transmission power, noise p.s.d. and bandwidth, which are same and fixed for all users. $\mathcal{O}_i$ is the set of transmitters that node i hears and it is determined by the routing algorithm. The cardinality of $\mathcal{O}_i$ is at most 2 in this study.

There is a target rate R_0 and the signal to noise ratio is required to achieve that rate is found using the Shannon capacity function $R_0 = W\log_2(1+\Gamma)$ bps. Let's define $e_0 = (2^{\frac{R_0}{W}}-1)\frac{N_oW}{P}$. The successful reception probability of node i (i.e. $\Pr(\sum_{n=1}^{|\mathcal{O}_i|} g_{n,i}h_{n,i} > e_0)$) for a given transmission order can be written as

$$P_s^i(\mathcal{O}_i) = \left(\prod_{n\in\mathcal{O}_i}\frac{1}{g_{n,i}}\right)\sum_{j\in\mathcal{O}_i}\frac{g_{j,i}e^{(-\frac{e_0}{g_{j,i}})}}{\prod_{k\in\mathcal{O}_i,k\neq j}(\frac{1}{g_{k,i}}-\frac{1}{g_{j,i}})} \tag{2}$$

The success probability of node 1 is one, since it is the source node. The transmitter set for node 1 is the empty set. Overall outage probability is the probability that at least one of the nodes on the path cannot decode because of outage and can be written as,

$$P_o(\mathcal{O}) = 1 - \prod_{i\in\mathcal{O}} P_s^i(\mathcal{O}_i) \tag{3}$$

It is a logical action to limit the number of nodes in a route. Because as the number of transmitters increase 1) Energy expenditure increases 2) Delay can increase 3) Interference increases.

III. Routing Solutions

Order of transmission starting from the source node, s, and ending at the destination node, d, is called as a *"route"*. Our purpose in this work is to find the cooperative route which minimizes the overall outage probability in equation (3). First, we will propose an optimal solution based on the Branch-and-Bound technique [11]. Then we will consider some suboptimal algorithms and compare their performances with the optimum. Some of the algorithms that we propose can only be implemented in a centralized way. Also we adapt the Minimum Power Cooperative Routing (MPCR) algorithm proposed in [10] to Minimum Outage Cooperative Routing (MOCR) algorithm and Improved Minimum Outage Cooperative Routing (IMOCR), which can be implemented distributively.

A. Centralized Routing Algorithms

1) Optimal Solution: Branch and Bound (BB) is a general method to find optimal solutions of combinatorial optimization problems [11]. In our problem we try to find the optimal order of transmissions which makes the overall outage probability minimum. A tree can be formed from all possible orders of transmissions (denoted as $\mathcal{B}$). For example let $\{1, 3, 5, N\}$ be a route. This can be branched into $N-4$ other possible routes as $\{1, 3, 5, 2, N\}$, $\{1, 3, 5, 4, N\}$, $\{1, 3, 5, 6, N\}$ etc. Let T_{max} be the maximum number of transmitters in a route. Depth of the search tree can be maximum T_{max}. Branch and Bound is an effective method of searching all possible solutions in the tree by eliminating at each step the solutions in the subtrees that are guaranteed to be non-optimal. This method is too complex for real-time wireless implementation but can serve as a benchmark for the practical suboptimal algorithms. This method requires three operations; *branching*, *bounding* and *pruning*. Branching is a recursive procedure to define a search tree whose nodes are candidate transmission orders for us. Second tool computes upper and lower bounds for the outage probability for each branch. The key idea of the BB method is: if the lower bound for some branch SET_x is greater than the upper bound for some other branch SET_y, then SET_x discarded from the search and this is called *pruning*. For a given ordered set of transmissions $\mathcal{O}$, the lower and upper bounds on outage probability are,

$$\texttt{LB}_{\mathcal{O}} = \begin{cases} 1 - \prod_{i \in \mathcal{O} \| N} P_s^i(\mathcal{O}_i) & \texttt{if } |\mathcal{O}| = T_{max} \\ 1 - \prod_{i \in \mathcal{O}} P_s^i(\mathcal{O}_i) & \texttt{else} \end{cases} \tag{4}$$

$$\texttt{UB}_{\mathcal{O}} = 1 - \prod_{i \in \mathcal{O} \| N} P_s^i(\mathcal{O}_i) \tag{5}$$

Here $|\mathcal{O}|$ is the cardinality of the set $\mathcal{O}$ members. If $|\mathcal{O}| < T_{max}$ the lower bound on outage probability is calculated by excluding the destination node N from the path (the actual overall outage probability is always higher than is bound). If $|\mathcal{O}| = T_{max}$ than the path is completed and instead of a lower bound, the exact outage probability is calculated. Upper bound on outage probability is computed assuming no other node will be added to the route and the nodes in $\mathcal{O}$ are the only transmitters. If there are T_{max} transmitters on the path, than the upper and lower bounds are equal. For example, consider the branch $\{1, 5, 2, N\}$, where N is the destination. If $|\mathcal{O}| = 3 < T_{max}$, then the lower bound of this branch is found by calculating the overall outage probability of the path $\{1, 5, 2\}$. The children of this branch in the tree always have higher outage probabilities. The overall outage probability of the path $\{1, 5, 2, N\}$ is be the upper bound of the branch. If $|O| = T_{max}$ then the upper and lower bounds become equal. After a series of branching , bounding and pruning operations only one branch (optimal one) remains and the search ends. The resulting procedure is called Routing Using Branch and Bound (RBB) and it is described as in Algorithm1.

2) Cooperative Routing with Cooperative Transmission (CR-CT): The centralized suboptimal algorithm, CR-CT, starts with order $\mathcal{O} = [1, N]$ and at each step adds a node to the order

Algorithm 1 Routing Using Branch and Bound (RBB)

Set $\mathcal{B} = \{\{1, N\}\}$ and calculate $\texttt{LB}_{\{1,N\}}$and $\texttt{UB}_{\{1,N\}}$
while true **do**
  find the best branch $b^* = \arg\max_{b \in \mathcal{B}} \{\texttt{LB}_b\}$
  if $\texttt{LB}_{b^*} = \texttt{UB}_{b^*}$ **then**
    $\texttt{bestbranch} = b^*$ and $\texttt{minoutage} = P_o(b^*)$
    search is ended, **return**
  else
    for $\forall\ n \notin b^*$ **do**
      Form new branch by adding n: $b = \{b^*, n\}$
      Calculate $\texttt{LB}_b$ and $\texttt{UB}_b$ using (4), (5)
      if $\nexists b' \in \mathcal{B}$ s.t. $\texttt{LB}_b > \texttt{UB}_{b'}$ **then**
        Add the branch to the set $\mathcal{B} = \mathcal{B} \cup b$
        if $\exists b' \in \mathcal{B}$ s.t. $\texttt{UB}_b < \texttt{LB}_{b'}$ **then**
          Prune branch b': $\mathcal{B} = \mathcal{B}/b'$
        end if
      end if
    end for
  end if
end while

resulting in minimum overall outage probability (3), with the T_{max} constraint. When there are i transmitters on the path, there are i alternative positions for the $i + 1^{st}$ transmitter. For example consider the path $[1, 5, 2, N]$, a 4^{th} transmitter (say, node x) can be added in three different positions resulting in paths $[1, x, 5, 2, N]$, $[1, 5, x, 2, N]$, $[1, 5, 2, x, N]$. An outage calculation is made for each candidate node and position. Considering this, we can say that the algorithm takes $\sum_{i=1}^{T_{max}} (N - i - 1)i$ outage calculations. This algorithm cannot have a distributed implementation because nodes 1 and N may not be able to hear each other at the beginning.

Algorithm 2 CR-CT: Cooperative Routing with Cooperative Transmission

1. $O = [1, N]$, $finish = 0$, $P_s = e^{-\frac{e_0}{g_{1,N}}}$, $P_{s,max} = P_s$
while $finish = 0$ or $|\mathcal{O}| < T_{max}$ **do**
  $i^* = 0$
  for all $i \in \mathcal{O}^c$ **do**
    for $j = 1$ to $|O| - 1$ **do**
      Set $\mathcal{O}' = [\mathcal{O}[1 : j], i, \mathcal{O}[j + 1 : |\mathcal{O}|]]$
      if $P_s(\mathcal{O}') > P_{s,max}$ **then**
        $i^* = i$
        $j^* = j$
        $P_{s,max} = P_s(\mathcal{O}')$
      end if
    end for
  end for
  if $i^* = 0$ **then**
    No improvement, $finish = 1$;
  else
    $\mathcal{O} = [\mathcal{O}[1 : j^*], i^*, \mathcal{O}[j^* + 1 : |\mathcal{O}|]]$
  end if
end while

B. Distributively Implementable Routing Algorithms

1) Noncooperative Routing with Cooperative Transmission (NCR-CT): The first algorithm named NCR-CT has a simple procedure to find a cooperative route. The algorithm (Algorithm 3) first finds the optimal noncooperative route based on a link cost metric, then cooperative transmission is performed on this route. Here, there is no path length constraint. We try to limit the path length by incorporating a hop count cost to the link cost. The algorithm computes costs for each node pairs as $C_{i,j} = \frac{e^0}{g_{i,j}} + C_h, \forall i, j$. Here $\frac{e^0}{g_{i,j}}$ is the approximate outage probability of the link between i and j. The constant C_h is the hop count cost. C_h must be chosen comparable to link outage probability. Bellman-Ford algorithm can be applied to find the minimum-cost route. [1]. Based on the noncooperative route, a cooperative route is formed by allowing each node to receive and combine signals from the two previous hops. This algorithm has $O(N^2)$ complexity.

Algorithm 3 NCR-CT: Noncooperative Routing with Cooperative Transmission

1. Distributed Bellman-Ford algorithm is applied with cost matrix $C_{i,j} = \frac{1}{g_{i,j}} + C_h, \forall i, j$ and an order $\mathcal{O}$ is found.
2. Each receiver uses the signals from the last two transmitters that transmits before it.

2) Minimum Outage Cooperative Routing (MOCR): First we adapt the Minimum Power Cooperative Routing (MPCR) algorithm proposed in [10] to Minimum Outage Cooperative Routing (MOCR) algorithm. First of all, here we assume that a node can combine only two last previous transmissions before it. This algorithm uses two types of transmissions: direct transmission (DT) and cooperative transmission (CT, Transmission with help of a relay node) due to the fact that the optimal route can be a combination of cooperative transmissions and point-to-point transmissions. Cooperative transmission's cost is bigger than direct transmission because one more link is constructed to employ the relay node an this can be seen from equations below.

$$C_{i,j} = \max\{1 - P_s^j(\{i\}) + C_h, \max_k\{1 - P_s^j(\{i,k\}) + 2C_h\}\} \quad \forall i, j \quad (6)$$

P_s^j can be computed using (2). Firstly, a node i checks its neighbors and for each neighbor j finds the best relay, $k^* = \arg\max_k\{1 - P_s^j(\{i,k\}) + 2C_h\}$, to reach j and forms a cooperative link. If direct transmission is better in terms of success probability ($P_s^j(\{i\})$), then a direct link is formed. This operation can be done in a distributed manner. Then using the distributed Bellman-Ford algorithm, the best end to end path is found from node 1 to N. The pseudo code is shown in Algorithm 4. In Figure 1, the second route is obtained by MOCR. Here the packets are routed through three cooperative links (1-8-12), (12-14-22), (22-9-25).

[1]If $C_h = 0$ then this route becomes the minimum outage noncooperative route [6]

Algorithm 4 MOCR: Minimum Outage Cooperative Routing

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for n = 1 to N do
    for ∀x ≠ n do
        Calculate 1 − P_s^x({n}) + C_h according to (2) as direct reward
        for ∀y ≠ n and y ≠ x do
            Calculate 1 − P_s^x({n, y}) + 2C_h according to (2) as cooperative reward
        end for
    end for
end for
Find max{1 − P_s^x({n}) + C_h, max_y{1 − P_s^x({n, y}) + 2C_h}} as the cost of the link (n, x)
if cooperative cost is less then
    Employ the certain relay y* to help the transmission over that hop
end if
Apply Distributed Bellman-Ford shortest path algorithm using the calculated link rewards.
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3) Improved Minimum Outage Cooperative Routing (IMOCR): This algorithm first uses MOCR. Then it finds the nodes on the path that are receiving from only one node (DT) and makes them receive from last two nodes (CT) in the route. For example in Figure 1, the nodes 14 and 9 can also hear from 8 and 12, respectively.

Algorithm 5 IMOCR: Improved Minimum Outage Cooperative Routing

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Apply MOCR algorithm.
Then find the nodes receiving from only one node and make them receive from two previous nodes in the route.
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IV. Numerical Evaluations

We consider a number of nodes uniformly located in a cellular area of radius 1000 meters. The source and destination nodes (1 and N) stand on the opposite sides of the circle. Each node has a fixed transmission power of 10mW. Noise power is -154 dB and the path loss model $31.5 + 35\log_{10} d$ dB, where d is the distance in meters. Rate requirement is 1Mbps and bandwidth is 1MHz, therefore received (effective) SNR constraint is equal to 1. The outage probability of node N is the performance metric.

In Figure 2 and Figure 3 we considered a 15-node network and use hop count cost of 1/30 and 1/50. The considered algorithms are IMOCR, MOCR, RBB, CR-CT and NCR-CT. For each algorithm, we generated 1000 random networks and calculated the overall outage probabilities with each of the algorithms. For each algorithm we obtained an outage vector of length 1000. We then divide the vector of each suboptimal algorithm to that of RBB, in order get the relative performance vector. We plot the empirical cumulative distribution functions of the relative performance vectors of each algorithm. For a fair comparison we do the following. We first run IMOCR and record the number of hops for each 1000 trials. Then we run RBB and CR-CT by setting T_{max} equal to those number of hops for the corresponding trials. Allowing the same number of hops facilitates a fairer comparison. The results in Figure 2 and Figure 3 show that RBB is optimal but IMOCR and CR-CT performed almost as good as the RBB. In 55 percent of the trials they find the optimal routes. IMOCR is the best of the suboptimal ones and it is within 5 percent of the optimal. The fact that it is distributively implementable, less complex and can perform better than CR-CT makes it a suitable candidate for real-time implementation. Please also note that IMOCR provides significant improvement with respect to MOCR.

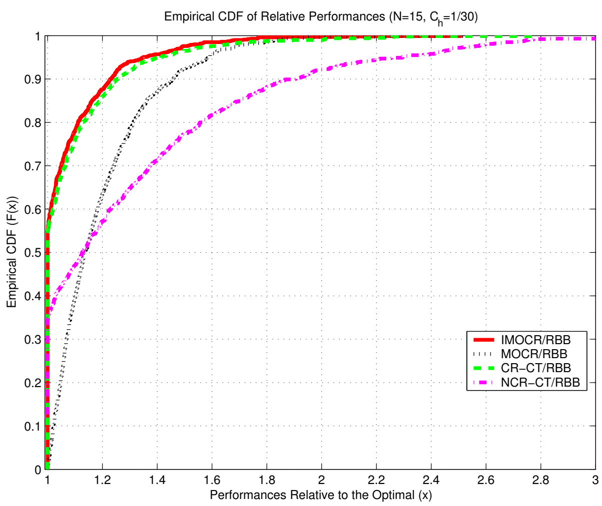

Fig. 2. Relative performance: N=15 and $C_h = 1/30$.

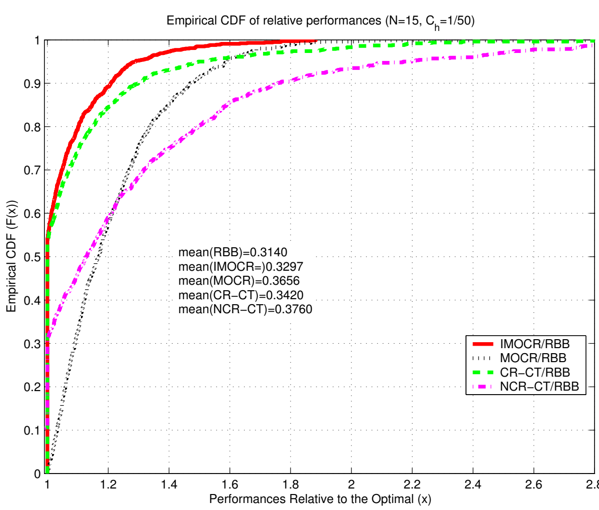

Fig. 3. Relative performance: N=15 and $C_h = 1/50$.

Figure 4 shows c.d.f. of outage probabilities of a 25-node network. We see that when the number of nodes increases mean outage probability decreases because the distance between nodes decreases. We observe around 40 percent decrease in outage probabilities.

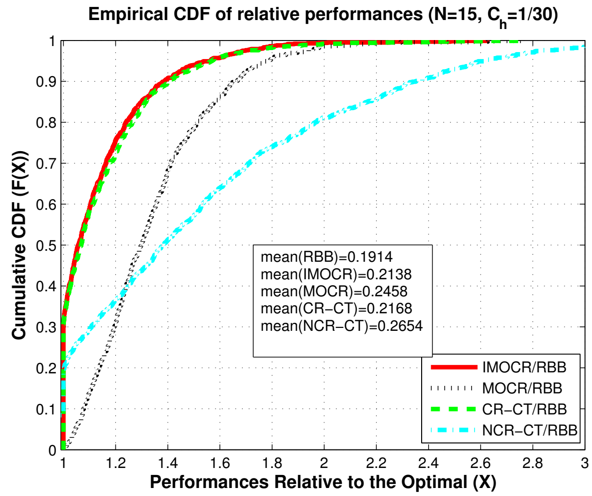

Fig. 4. Relative performance: N=25 and $C_h = 1/30$.

V. Conclusions

In this paper we worked on cooperative unicast algorithms with minimum outage. The results show that the IMOCR algorithm can be implemented in a distributed way and performs within $\%5$ of the optimal solution. This makes it a good candidate for real-time implementation. As future work, we will consider finding minimum power routes that satisfy a certain target outage probability. Interference and transmission scheduling will be also jointly considered.

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