Routing with Mutual Information Accumulation in Energy-Limited Wireless Networks

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***Abstract*—We consider the problem of minimum energy unicast routing in the presence of idealistic rateless codes. The nodes on the path are able to accumulate mutual information from the transmissions of the previous nodes on the path. We first consider the case of nodes with unlimited energy and propose an algorithm that outperforms a method proposed in the recent literature. We then consider the case with limited-energy nodes. We prove by counter examples that some properties that hold in the unlimited energy case, do not hold anymore in the limited energy case. Next we describe a suboptimal algorithm and compare its performance with the optimal solution.**

I. Introduction

In the past decade, there has been a sustained effort dedicated on cooperative communications and its noticeable gains with respect to the traditional wireless communication systems. The majority of the previous work on cooperative communications has focused on energy accumulation[6]. In that model the receiver combines the signals coming from different paths using techniques such as maximal ratio combining and the receiver is able to decode the message if the sum of individual SNRs exceed a threshold level. In this case each transmitter has to transmit the same bits using the same modulation and coding.

Rateless Codes [2] [7] facilitate accumulation of codewords (instead of energy) at the receiver. Basically, the transmitter divides the available information into K blocks, and at each time randomly chooses and XORs a subset of the blocks, such that the receiver will be able to decode the original message when it accumulates a sufficient number of coded packets. In a multihop scenario, as soon as a relay node decodes the message, it starts to retransmit the message using fountain encoding. A node on the path can accumulate coded packets from the previous hop transmissions, which improves the energy-efficiency. In the literature usually idealistic rateless codes are assumed, where the nodes can accumulate mutual information instead of packets, and the receiver can decode the message whenever the amount of received mutual information from previous transmissions exceeds the message size. In [1], [3], the authors showed that in a high SNR regime the information accumulation technique works with lower energy expenditure and time latency than classical energy accumulation techniques.

The focus of this study is energy efficient design of wireless networks, which is gaining renewed attention in the light of the recent Green ICT movement, that motivated new projects such as the E-CROPS project under the CHIST-ERA program.

This study is on energy efficient transmission for wireless networks with a single source and a single destination. In the rest, we first present a study on the unlimited energy case and present an efficient optimal solution, which is based on the results of [9]. Then we propose a heuristic based on Dijkstra's algorithm. Then we analyze the same problem for the limited-energy case. We present an analysis based on counter examples, that shows that this is an interesting and hard problem, and some important properties that hold for unlimited energy do not hold anymore for the limited energy case. A heuristic method will also be provided.

The studies [9], [4] are the most relevant. In [3] Draper et. al. proposed a method for finding the optimal path by solving a Linear Programming (LP) problem for each subset and order of the nodes which requires the computation of $\sum_{k=0}^{N} \binom{N}{k}$ LPs, where n is the number of relays. In [9] the authors showed that for unicast routing problem a greedy algorithm can be applied and showed the complexity of finding the optimal path by using greedy algorithm is 2^N. In [4] Draper et al. gave a heuristic method for the optimal path in which the algorithm calculates a polynomial number of LPs. In [5] the authors considered the energy minimization problem with variable transmission duration and times, and they proposed an algorithm that finds a suboptimal route, transmission times and powers for each node.

In Section II the system model will be described in detail. Later in Section III the focus is on the optimal path for the unlimited energy case. First a short study on the optimal path will be given and later in that section we will suggest a heuristic method which has a polynomial complexity of $O(N^3)$. Its performance will be evaluated in Section V, where we will see that the difference in transmission time for 98 % of the samples is less than 3 % with respect to the optimal. In Section IV we will focus on the energy limited case. First we give a study on the optimal path then a heuristic method will be proposed and its performance will be shown in the Section V. The complexity of the heuristic method is $O(N^2)$. In Section V the simulation model will be described in detail and the results will be presented. Finally, in the Sec. VI we conclude the paper and highlight some directions for future work.

II. System Model

Similar to [3] we consider a network consisting N+2 nodes which includes a source, a destination and N potential relays like in Figure 3. The channel power gain between each pair (i,j) is denoted by $h_{i,j}$. Channel conditions are assumed to be fixed throughout the end-to-end transmission. The channel capacity between two nodes i,j is denoted by $C_{i,j}$ (bits/sec/Hz). If node i transmits for Δt seconds the amount of information which node j will gather is $C_{i,j}\Delta t$ bits/Hz. We assume that the transmission power is equal and fixed for all nodes, therefore minimizing energy and delay are equivalent objectives.

In this study we focus on unicast transmission where the source has a packet to be delivered to the destination and finding a route with minimum energy expenditure (or equivalently delay) is the issue of this study. Here we assume that the network has just one free channel (of bandwidth W Hz) for transmission. So just one node can transmit at each time and the others should be kept silent. During the transmission undecoded nodes keep track of the transmissions until they gather B bits/Hz of data from previous hop transmissions. For example if the routing path is $\overline{\pi} = [1 = \pi_1, \pi_2, \ldots, \pi_j, \ldots, N+2]$, node π_j decodes the message at the end of $j-1^{st}$ stage if the following formula is valid,

$$\sum_{i=1}^{j-1} A_{\pi_i} C_{\pi_i,\pi_j} \geq B \qquad (1)$$

where A_i is the duration of transmission of node i. The problem becomes finding the optimal path that starts with the source and ends at the destination, and finding the time allocation on that path that results in minimum total energy, subject to rate constraints 1 for all nodes on the path. Although this mutual information accumulation assumption is based on idealistic rateless codes, it can easily be generalized to the practical cases by multiplying B by (1+ϵ) where ϵ captures, the additional time/energy expenditure due to non-idealities.

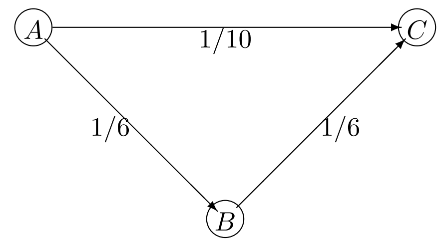

Fig. 1. A three node topology that shows the advantages our modification with respect to the original Dijkstra's algorithm. The weights denote the channel capacity of each link. Roughly, to send 1 bit, the number of transmissions needed on path A,C is 1/(1/10)=10. On the other hand, the number of transmissions needed on path A,B,C is: $6 + (1 - \frac{6}{10}) \times 6 = 8.4$.

III. Energy Efficient Routing with Unlimited Energy at Nodes

A. Finding the optimal path

In [9] the authors showed that the optimal path should satisfy the two conditions below.

1) Just one node transmits during each time slot. (A time slot is defined as the duration between two consequent nodes decoding the message.)
2) Given the optimal set of transmitting nodes, each transmitter in this set starts to transmit as soon as it decodes.

Based on these results the problem of finding the optimal route and transmission times, reduces to finding the optimal set of transmitting nodes, which has a complexity of 2^N.

B. Heuristic method

Dijkstra's well known shortest path construction algorithm using link costs could be naively applied here, as a heuristic, by taking as weights the mutual information accumulated from a single link (disregarding the previous hop transmissions). With such link-based metrics performance in some cases becomes far from the optimal. In this paper we want to modify Dijkstra's algorithm in a way that improves its efficiency, while keeping its desirable polynomial complexity. The following example illustrates our motivation. Consider the three-node network in Figure 1. The number written on each link is the capacity (number of bits per transmission) of that link.

In this network, simply applying Dijkstra's algorithm taking link weights as the reciprocal of link capacity (corresponding to the number of transmissions per bit), without considering mutual information accumulation suggests path A,C for the completion of transmission in the shortest amount of time (with the smallest number of transmissions). But, taking into account mutual information accumulation, we find that path A,B,C is much better.

The proposed algorithm works with a parameter k, where k is the number of nodes on the path from whom each node on the path can accumulate mutual information. In other words, each node on the path can accumulate mutual information from the last k nodes on the path.

1) Proposed Suboptimal Algorithm (Heuristic-U): This algorithm is based on Dijkstra's algorithm but with a difference. In the classical Dijkstra's algorithm in each stage the cost of a unvisited nodes are calculated by adding the cost-to-go of a visited node with the link cost between the visited and unvisited node. In our modification, link cost between a visited and unvisited node is calculated as the residual mutual information of the unvisited node divided by the achievable rate of the link between the visited and unvisited nodes. We also add a parameter k, which denotes the number of previous hops from which a node can accumulate mutual information.If we set $k = 1$, the heuristic reduces to the on proposed in [9]. The proposed algorithm is described in the pseudocode 1 below.

The performance of this algorithm will be evaluated in Section V. The complexity of the algorithm is $O(N^3)$. The complexity can be reduced if we the parameter k is reduced. If we set $k = 1$ the algorithm becomes the original Dijkstra's algorithm.

Algorithm 1 Proposed Suboptimal Routing Algorithm for the Unlimited Energy Case (Heuristic-U)

Define T_i as the minimum access time of node i. And set $T_{source} = 0$ and the rest set to ∞ as the initial value
Define set of unchecked nodes Π_u and of the checked nodes Π_c
Set $\Pi_c = \emptyset$ and set $\Pi_u = \Pi$.
For all nodes, set `access path of node i` = $\emptyset$
while `destination` $\in \Pi_u$ **do**
  Choose a new node (node n) in Π_u as $\arg\min_{i \in \Pi_u} \{T_i\}$
  if n=destination **then**
    Set $\Pi_u = \Pi_u /$`destination` and $\Pi_c = \Pi_c \cup$ `destination`
  else
    Set $\Pi_u = \Pi_u / n$ and $\Pi_c = \Pi_c \cup n$
    Follow the access path of node n from the source and keep track of the amount of information the other nodes gather from the transmission of last `k` nodes on the path.
    Set t_i for $i \in \Pi_u$ to $T_n +$ (`remaining info`$/C_{n,i}$)
    if $t_i < T_i$ **then**
      $T_i = t_i$
      set `access path of node i` = `access path of node n` $\cup\, n$
    end if
  end if
end while

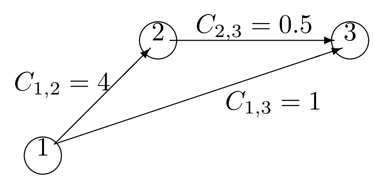

Fig. 2. A three node topology for Remark 1

IV. Energy Efficient Routing with Energy-limited Nodes

A. Finding the optimal path

In practice most networks suffer from energy limitations, which is directly related to the network lifetime duration. In this section we present a study on limited energy case and analyze whether the greedy algorithm in [9] holds for this case. In the end, a suboptimal algorithm will be described and its performance will be provided in Section V.

Draper et al. [3] presented an algorithm for optimal scheduling for minimum energy transmission which need to solve N! linear programs (N is the number of relays). As it was mentioned before in [9] it was proved that for an unlimited-energy case, given the optimal set of transmitting nodes, the optimal transmission order and durations can be found using a greedy algorithm. The following examples show that the greedy algorithm of [9] cannot be generalized for the limited energy case.

Remark 1: Even if a node is in the optimal set of transmitting nodes, it does not necessarily start to transmit as soon as it decodes.

Proof is by a counter example. Consider the topology in Figure 2. The numbers on the links denote the achievable rates. Assume that the destination has to accumulate 10 units of mutual information and node 1 has 8 units of initial energy. Node 1 starts to transmit and node 2 decodes at time 2.5. If node 2 starts to transmit, the total transmission duration becomes 17.5 time units . On the other hand if node 1 continues transmitting, it runs out of energy at time 8, and node 2 transmits for 4 units of time, which results in a total duration of 12 time units. This proves the correctness of the Remark. It also implies that, even if we are given the optimal set of nodes, we may still have to check every possible transmission order.

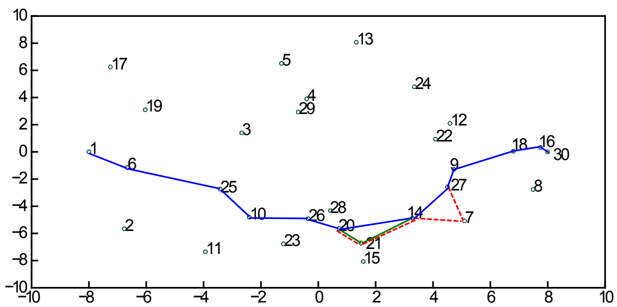

Fig. 3. A sample network with 28 relays. Where node 1 is source and node 30 is the destination. The blue, green, and the dashed red lines are the chosen paths computed for $k = 1, 5, \infty$ respectively

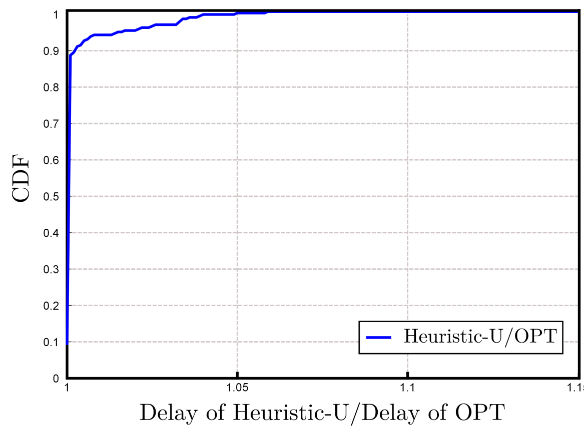

Fig. 4. Nodes with unlimited energy: Cumulative Distribution Function plot of the ratio of transmission time of Heuristic-U to OPT over all randomly generated example cases.

In [9] it was shown for unlimited-energy case that between each two events (an event is a decoding instant) just one node transmits in the optimal scheduling. In this work we define an event as a time when something happens in the network (which could be a new node decoding the message or a node running out of its energy). Our next remark is as follows,

Remark 2: In the energy-limited case, between two events more than one node may transmit in the optimal solution.

The proof is given in Appendix A.

These two remarks show that in order to find the optimal routing 1) 2^N possible transmitting sets may need to be checked as in [9], 2) We also need to run a linear program for each order (differently than [9]).

B. A Heuristic Method (Heuristic-L)

Here we present a heuristic method with a complexity of $O(N^2)$. A similar algorithm was suggested in [8] for unlimited energy case. In this algorithm the sender continues the transmission until the first event happens. At every event (whether a new node decoding the packet, or the transmitting node running out of energy), the algorithm choose the node (among the nodes that have already decoded the packet and have energy) that has the best achievable rate to the destination.

V. Simulation Results

In this section the simulations will be described in detail first, then the numerical results will be presented. Here we consider a 2D network, which consists of a source and a destination and N relays which are distributed uniformly random inside a circle with radius 10 and center (0,0). Source is located at point (-8,0) and the destination is at point (8,0) (Figure 3).

In order to understand the basics of the problem, we ignore the effect of fading, or channel variations, and assume constant channel capacities. We set the message size to an arbitrary value, 10 bits. We compute channel capacity values as $C_{ij} = \log_2(1 + \frac{1}{d_{ij}^2})$.

In the first set of simulations, we wish to gauge the performance of the heuristic method which has been proposed for the unlimited energy case. Keeping the locations of the source and destination fixed, the positions of relay nodes are varied over 200 instances. The number of relays is 18.The plot of cumulative distribution function (CDF) of the ratio of transmission time of the heuristic method over the optimal route has been brought in Figure 4.

As it is seen in the plot in more than 98% of the samples the difference between the transmission times is less than 3%. As it was mentioned in the heuristic method we can set different value for k (track the information each node gathered during the last k transmission of that path) for the algorithm. In the Figure 5 the result of the transmission time of the algorithm for different values of k is compared. In there we assume the number of the nodes is 30. Cumulative distribution of the ratio of transmission time corresponding to k=1,2,5 to transmission time corresponding to unlimited k is shown in Figure 5.

As we see from the Figure, when we set k=5, the performance is almost optimal. The case k=1, corresponds to the heuristic algorithm in [9].

Finally to check the performance of the heuristic method (Heuristic-L) for the case of energy limited nodes, the plot of the CDF of the ratio of transmission time of Heuristic-L over OPT (which is found by an exhaustive search and LP) over 100 instances is shown in Figure 6. Number of potential relays in the network is 10. The plot shows that with 90% probability the heuristic a performance which is within 7% of the optimal. Considering these results and the simplicity of Heuristic-L, it proves to be a quite effective scheme.

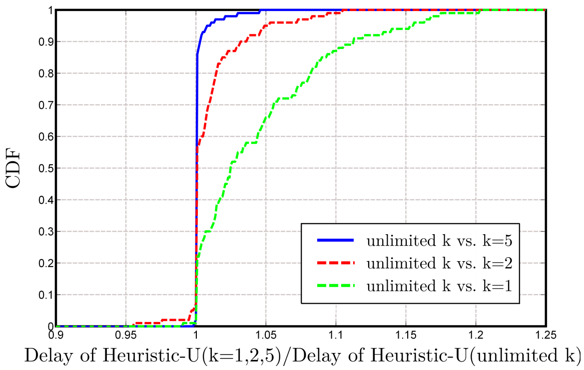

Fig. 5. Nodes with unlimited energy: CDF plot of the ratio of transmission time of Heuristic-U with k=1,2,5 to Heuristic-U with unlimited k, over all randomly generated example cases.

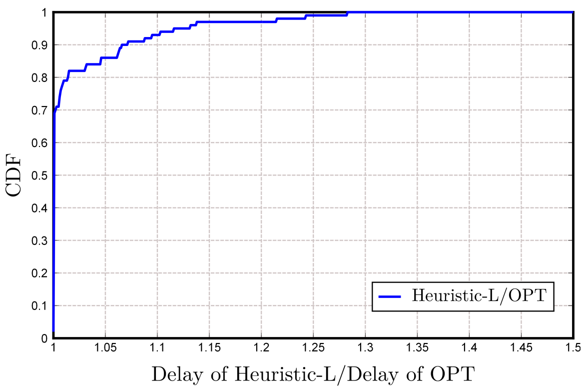

Fig. 6. Limited Energy Case: The Cumulative Distribution Function of the ratio of transmission time of the heuristic method over the optimal schedule (12 nodes)

VI. Conclusions

We considered routing with the minimum number of transmissions (equivalently, minimum energy) on a cooperative wireless network where nodes have the ability to perform mutual information accumulation . For the case of unlimited energy at nodes, we proposed a heuristic which is based on Dijkstra's algorithm. Numerical evaluations show that it performs very close to optimal, and outperforms a related heuristic proposed in recent literature. Next we considered the case where nodes have limited energy. By counterexamples we proved the greedy policy for the optimal set which is proposed before for unlimited energy case fails in the limited energy case. The optimal solution involves enumerating all possible subsets (i.e. set of transmitting nodes) of the set of nodes, and running a Linear Programming solution for each of them. We then exhibited a heuristic for this case which performs very close to optimal.

VII. Appendix

Here we want to show that in the optimal scheduling it is possible that more that one node transmit between two in sequence events. Let's assume 3 nodes participate in optimal transmission. So the below equations should be satisfied

$$\begin{aligned} \min \quad & T_1 + T_2 + T_3 \\ \text{s. t.} \quad & T_1 \times C_{1,2} \geq B \\ & T_1 \times C_{1,3} + T_2 * C_{2,3} \geq B \\ & T_1 \times C_{1,d} + T_2 * C_{2,d} + T_3 * C_{3,d} = B \\ & T_1 \leq E_1, T_2 \leq E_2, T_3 \leq E_3 \end{aligned}$$

By applying the third constraint the below equation will be derived

$$T_3 = \frac{1}{C_{3,d}} * (B - T_1 * C_{1,d} - T_2 * C_{2,d}) \qquad (2)$$

If we rewrite the optimization equation with respect to the equation 2 the below formula will be derived

$$\begin{aligned} \min \quad & \left(1 - \frac{C_{1,d}}{C_{3,d}}\right) \times T_1 + \left(1 - \frac{C_{2,d}}{C_{3,d}}\right) \times T_2 \\ \text{s.t.} \quad & T_1 \times C_{1,2} \geq B \\ & T_1 \times C_{1,3} + T_2 \times C_{2,3} \geq B \\ & T_1 \times C_{1,d} + T_2 \times C_{2,d} \geq B - E_3 \times C_{3,d} \\ & T_1 \leq E_1, T_2 \leq E_2 \end{aligned}$$

In this netwok the possible events are: node 1 or 2 finishes its energy (which is showed by line4, 5 respectively), the energy of node 3 finishes (which is not happens during the transmission of node 1 and node 2 so it is not shown in the fig 7). Or node 2 or node 3 decode the packet (which is showed by lines 1 an 2 respectively) and finally the destination decode the packet which is showed by the colored area (feasible region). As it is seen in the above constraints, all of them except the third constraint declare an event in the network.

Fig, 7 shows one of the possible feasible regions. The optimal time schedule is the first point of feasible region that the line corresponding to the minimum function will touch as it is moving upward. In the Figure 7 if we move up the min function line, point A is the first point of feasible region it will touch. So it is the optimal solution.

Now if we assume that between each two in sequence events just one node will transmit, we start from the origin and in each step we can move right or up until it touch one of the lines 1,2,4 or 5. But with this process point A is not accessible. So, for reaching node A in at least during one of the two in sequence events more than one node transmit.

Acknowledgement

This research was supported by The Scientific and Technological Research Council of Turkey, via grants 110E256 and 112E175. The latter grant provides the local funding for the Chist-ERA E-CROPS project.

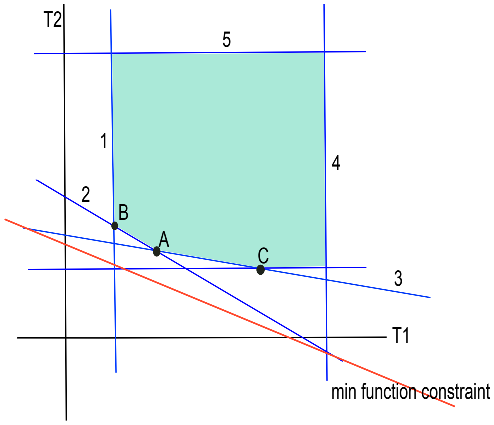

Fig. 7. Example of a feasible region to illustrate the discussion in the Appendix.

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