Fair Beam Allocation in Millimeter-Wave Multiuser Transmission

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***Abstract*—This paper addresses the problem of proportional fair beam allocation in millimeter wave (mmWave) switched-beam based systems. Working at the mmWave band facilitates using a massive number of antenna elements at the base station (BS). Usage of beamforming in large antenna arrays provides high directivity and increased SINR at the receivers. In this setting intelligent beam allocation over multiple time slots is required for fair rate allocation to users. Activating multiple beams simultaneously requires an algorithm that takes interbeam interference into account. We formulate the proportional fair beam allocation as a mixed integer nonlinear programming with an objective of logarithmic sum of average received rates. As for received rates at each time slot, Shannon capacity is used, taking the inter-beam interference into account. We also propose a near-MINLP-based solution as our interference-aware fair beam allocation algorithm. Numerical evaluation results reveal that proposed proportional fair beam allocation algorithm performs very close to the MINLP-based solution and performs much better than the considered benchmark algorithms.**

I. INTRODUCTION

In recent years, as data traffic is continuously increasing due to the bandwidth demanding trends including live video streaming, VoIP and social media usage correlated with ever growing number of mobile devices, the need for more wireless bandwidth becomes much more crucial. Considering also the bandwidth requirement challenges that will arise as 5G mobile communication becomes available, alternative innovations or techniques to the ones used in existing wireless communication should be introduced. To overcome this higher bandwidth requirement, using the millimeter wave (mmWave) frequencies which offer a wide bandwidth is a good alternative especially for next generation wireless networks such as 5G [1], [2].

Thanks to the small wavelengths at mmWave band, it is possible to pack large antenna arrays in smaller dimensions. This concept is identified as massive MIMO [3] and provides even higher directivity as well as signal to noise ratio (SNR). Directivity term is used as a measure of how concentrated a beam is in terms of power density compared to the isotropic antenna with same radiation power as in [4] for this study. Although it promises such a wide frequency spectrum, mmWave band has its own issues to be resolved. Studying at mmWave frequencies, the main problem becomes high path loss [1]. To overcome this issue and be able to use this band effectively, beamforming is a critical technology which offers high directivity by utilizing antenna arrays [5]. Such directivity compensates this path loss and makes mmWave a strong candidate for future wireless communication bandwidth. There are mainly two types of beamforming techniques, which are analog and digital beamforming as well as hybrid beamforming technique which is the mixture of two [6], [7], [8]. Digital beamforming is a technique that is achieved by adjusting the signal properties digitally in baseband [9], whereas analog beamforming is achieved by making use of phase shifters on antennas [10]. For the wireless communication systems based on beamforming, beam allocation to users is a considerable problem. However, an efficient beam allocation solution should take the interbeam (i.e sidelobe) interference into account. In this study, an analog beamforming scheme with directional, fixed angle beams [11], which are attained by Butler Method [12] is used.

In this paper, the problem of proportional fair beam allocation to users in a multi-user downlink transmission system with multiple beams is addressed. Beam allocation was previously studied in [4] which aims maximizing the total rate within the regarding system. The authors in [4] claimed in their work that service ratio (the number of users transmitted or number of beams activated in a single allocation) is an important parameter for fairness and a constraint for service ratio could be included in the optimization problem. However, they did not take into account the sidelobe interference and its effects on a fair beam allocation. We show in this work that, instead of a service ratio constraint, a proportional fair beam allocation can be performed over multiple time slots in order for each user to receive a fair share of rates provided by the BS. To be able to come to that result, this study develops a MINLP-Based solution and an near-optimal algorithm with less complexity. In addition to MINLP-based solution and the developed algorithm two benchmark algorithms are introduced and the performance evaluations of these four methods are discussed in this study.

The remainder of this paper is organized as follows. Section II explains the developed system model to solve the beam allocation problem. Section III gives the MINLP problem formulation for the system model. Section IV describes the proposed algorithm to solve the problem with less complexity than MINLP solution and defines a benchmark to compare it with the algorithm and finally the numerical results are presented in Section V.

II. SYSTEM MODEL

In this study, we assume a Base Station(BS) equipped with N array of antenna elements located at the center of the cell equally spaced and transmitting to a group of K users located randomly in the coverage area, as shown in Figure 1(a). Base station creates beams with equal beamwidth each covering a certain angular region as shown in Figure 1 (b) and thus makes the switched beam architecture possible.

Let parameter d_k be the distance of the kth user to BS, α be a constant denoting path loss exponent and θ be the angular position of the receiver. Parameter $g_{k,n}$ denotes the power delivered by the beam n to user k with respect to the unit power transmitted for this beam by the BS. Let $g_{k,n}$ be formulated as Equation (1)

$$g_{k,n} = D_n(\theta_k) d_k^{-\alpha} \quad \forall k, n, \tag{1}$$

where $D_n(\theta_k)$ is the beam directivity variable which is used as a measure of how concentrated a beam is in terms of power density compared to the isotropic antenna with same radiation power and formulated as Equation (2) [4].

$$D_n(\theta) = \frac{2(AF_n(\theta))^2}{\int\limits_0^{\pi} (AF_n(\psi))^2 \sin(\psi) d\psi} \tag{2}$$

$AF_n(\theta)$ in Equation (2) is the array factor of the beam n with respect to angle θ and formulated as Equation (3) [4]

$$AF_n(\theta) = \frac{\sin(0.5N\pi\cos\theta - \beta_n)}{0.5N\pi\cos\theta - \beta_n} \tag{3}$$

where

$$\beta_n = \zeta_n \pi, \tag{4}$$

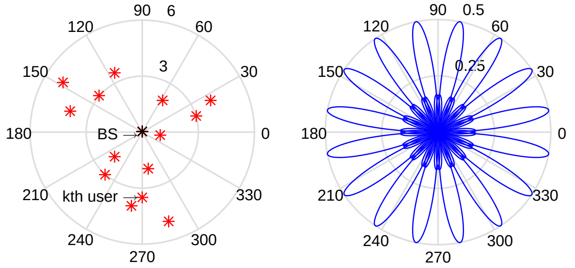
and

$$\zeta_n = -\frac{N+1}{2} + n. \tag{5}$$

Let P^{max} be the total transmit power that BS has for all beams. This total power is equally shared among the transmitting beams. K_{total} denotes the total number of nodes transmitted (or beams transmitting) where $K_{total} \in 1,..,K$. Let $P_{user} = \frac{P^{max}}{K_{total}}$ be the transmit power per user. Binary variable $c_{k,n} \in \{0,1\}$ denotes the beam allocation status. If beam n is allocated to user k then $c_{k,n} = 1$, if not, then $c_{k,n} = 0$. Based on these parameters and variables, $R^t_{k,n}(\mathbf{c}, K_{total})$ denotes the achievable rate for user k at beam n in time slot t and expressed as in Equation (6).

$$R^t_{k,n}(\mathbf{c}, K_{total}) = \log_2\left(1 + \frac{\frac{P^{max}}{K_{total}} \times g_{k,n}}{\sigma^2 + \sum\limits_{j=1, j\neq k}^{K} \sum\limits_{m=1}^{N} c_{j,m} \times \frac{P^{max}}{K_{total}} \times g_{k,m}}\right), \quad \forall n,k \tag{6}$$

Our goal is to maximize the proportional fairness of long term received rates of users. Let us define $\overline{R}_k^t$ as the average rate for user k up to the time slot t. The measure of proportional fairness is the sum of logarithms of average received rates $\sum_k \log\{\overline{R}_k^t\}$ [13]. Average rate is updated at each time slot as Equation (7)



(a) (b)

Fig. 1. (a) An example scheme consisting of one BS and K users where K=14 (b) Beam patterns, N=16

$$\overline{R}_k^{t+1} = \gamma \overline{R}_{k'}^t + (1-\gamma) \sum_{n=1}^{N} c_{k,n} R^t_{k,n}(\mathbf{c}, K_{total}), \tag{7}$$

where γ is a constant close to 1.

Proportional fairness is a suitable measure for both improving throughput and doing it in a fair way. Another measure of fairness in the literature is Jain's fairness index [16]. This metric can be formulated as $\mathcal{J}(R_1, ..., R_K) = \frac{\left(\sum_{k=1}^{K} R_k\right)^2}{K \sum_{k=1}^{K} R_k^2}$. Maximum value of this metric is equal to 1 and it is achieved if all rates are equal. Hence, this metric does not encourage improving the total throughput and it is limited by the user with the worst channel condition. This is the main reason of choosing log-sum rate instead of Jain's fairness metric.

III. PROBLEM FORMULATION

In this study, we formulate an optimization problem to schedule beam allocation to users. As in [13] the above-defined log-sum rate objective can be closely approximated by a weighted sum rate, where the weights are the inverse of average received rates. We model the optimization problem below, which is to be solved separately at each time slot,

$$\max_{\mathbf{c}, K_{total}} \left\{ U(\mathbf{c}, K_{total}) = \sum_{k=1}^{K} \frac{\sum\limits_{n=1}^{N} c_{k,n} \times R^t_{k,n}(\mathbf{c}, K_{total})}{\overline{R}_k^t} \right\} \tag{8}$$

subject to

$$\sum_{n=1}^{N} c_{k,n} \leq 1, \quad \forall k \tag{9}$$

$$\sum_{k=1}^{K} c_{k,n} \leq 1, \quad \forall n \tag{10}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n} = K_{total} \tag{11}$$

$$P_{user} \times K_{total} = P^{max}, \quad \forall k, n \tag{12}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n} \geq K_{min} \tag{13}$$

Objective (8) defines the weighted sum of the rates of all users as the objective function. Inequality (9) indicates that a user can only be allocated to one beam and Inequality (10) indicates that a beam can only be allocated to one user. Equation (11) enforces that K_{total} users are transmitted, which is an optimization variable. Equation (12) defines the transmission power for each user. Inequality (13) is the constraint defining a minimum service ratio (minimum number of users to be served) at each time slot.

IV. PROPOSED SOLUTIONS

The above optimization model is nonlinear, with continuous and integer variables. Therefore it can be considered as a Mixed Integer Nonlinear Program (MINLP). We can solve this problem using the BARON solver in the GAMS software package, in order to obtain the MINLP-based solution of beam allocation.

As for suboptimal solutions we consider a benchmark algorithm and also propose a near MINLP-based solution for beam allocation algorithm. Our benchmark algorithm (Proportional Fair Beam Allocation (PFBA)) is inspired by the suboptimal algorithm in [4]. This algorithm performs beam allocation disregarding the interference. We revised this algorithm with the aim of proportional fairness. The algorithm scans the users one by one. For each user the best beam (with the highest directivity) is found. If the beam is already allocated to another user, then the users are compared according to the metric $\frac{1}{R_k^t}\log_2\left(1+\frac{P^{max}\times g_{k,n}}{\sigma^2}\right)$, $\forall n,k$. If the metric for the current user is greater than the originally allocated user, then a reallocation occurs. The allocation won't change, otherwise. This is a quite simple algorithm with a complexity of $O(NK)$. On the other hand, it does not take into account the interference, which significantly degrades the performance, as will be seen in the simulation results.

Our proposed algorithm is called Interference-Aware Proportional Fair Beam Allocation (IPFBA). Algorithm 1 shows the pseudocode of IPFBA. Line 1 is the initialization step. At each step (Lines 2-15) the algorithm tries to allocate one free beam to one free user, in a way that improves the utility function (8) most. Free beams/users are the ones that have not been used/served yet. Once a beam is allocated to a user, both are excluded from the set of free beams and users, respectively. Each newly paired beam and user adds to the total utility, on the other hand, it creates extra interference to the other users and decreases the power per beam (P_{user}). At some point adding one more beam-user pair does not improve the total utility, at which point the algorithm terminates. In its current form, the proposed IPFBA Algorithm requires knowing the channel gain from each beam to each user. A more practical version would use the channel gains of only two or four beams that are closest to the angular position of the user.

At each iteration of the algorithm each free user-beam pair is checked. Besides, in order to calculate the total utility in Line 6, interbeam interferences have to be calculated. In the extreme case the algorithm may schedule all users, which means K iteration. Therefore the worst case complexity of this algorithm is $O(K^3N^2)$. If for each user only the best two or four beams are checked, then the complexity reduces to $O(K^3N)$.

Algorithm 1 Interference Aware Proportional Fair Beam Allocation (IPFBA)

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 1: Initialize $\mathcal{N}' = \mathcal{N} = \{1,2,...,N\}$, $\mathcal{K}' = \mathcal{K} = \{1,2,...,K\}$, $c_{k,n} = 0, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$ , $U^{max} = -\infty$, $K_{total} = 0$
 2: while there is improvement in total utility do
 3:     $k^* = 0, n^* = 0$
 4:     for $k \in \mathcal{K}', n \in \mathcal{N}'$ do
 5:         $\mathbf{c}' = \mathbf{c}$, $c_{k,n} = 1$
 6:         Calculate $U(\mathbf{c}', K_{total}+1)$
 7:         if $U(\mathbf{c}', K_{total}+1) > U^{max}$ then
 8:             $k^* = k, n^* = n$
 9:             $U^{max} = U(\mathbf{c}', K_{total}+1)$
10:         end if
11:     end for
12:     if $k^*, n^* > 0$ then
13:         $c_{k^*,n^*} = 1$
14:     end if
15: end while
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V. NUMERICAL RESULTS

As for the simulation model, we consider K users and N beams. The simulation lasts for $T = 100$ time slots. Each solution method mentioned above is run for each time slot, while the average user received rates are updated according to (7). The parameter γ is taken to be 0.9. At the end of the simulation the logarithms of the resulting average rates of each user are taken and summed. Path loss (in dB) is $20\log_{10}\left(\frac{4\pi}{\lambda}\right)+10\alpha\log_{10}(d)+\Phi$, where d is the user distance in meters and α is the path loss exponent, which is taken as 2.7. Parameter Φ is the log-normal shadowing, which is a Gaussian random variable with a standard deviation of 9.6 dB [14]. Noise power spectral density is -174 dBm and the system bandwidth is 800 MHz.

We first evaluate the effect of the service ratio constraint. We take $N = 8$ beams and distribute $K = 4$ users uniformly in a circle of radius $D_{max} = 200$ meters. Path loss of each user is fixed throughout the simulation. Figure 2 shows the

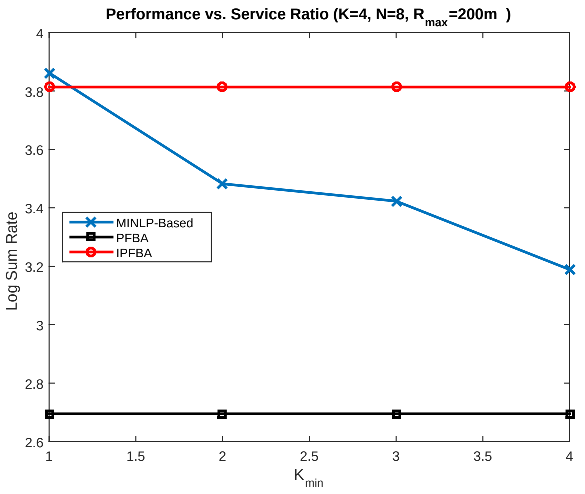

Fig. 2. Performance (log-sum rate) vs Minimum Service Ratio Constraint (K_{min}) ($K = 4, N = 8, D_{max} = 200$ meters).

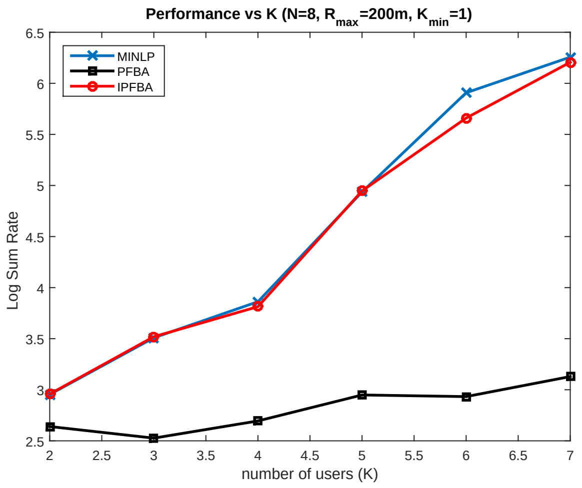

Fig. 3. Performance vs Number of Users ($N = 8, D_{max} = 200$ meters, $K_{min} = 1$).

log-sum rate performance of the three methods as a function of the minimum service ratio constraint, K_{min}. The K_{min} is effective only for the MINLP-based method. As the plot reveals, for $K_{min} = 1$, the MINLP based solution performs best. However as K_{min} increases enforcing the activation of more beams increases the interbeam interference and decreases the proportional fairness metric. This proves that instead of enforcing a service ratio, fair beam allocation can be performed over multiple time slots in order to maintain fairness.

Figure 3 plots the log-sum rate versus number of users, for a system of $N = 8$ beams. The results make it clear that the proposed IPFBA algorithm performs almost optimally compared to MINLP-based solution. The benchmark PFBA algorithm on the other hand disregards side lobe interference in beam allocation, which seriously decreases the throughput and proportional fairness performance.

Figure 4 shows the performance of the suboptimal algorithms for a larger number of users and beams. Number of beams is $N = 32$ and the number of users varies from $K = 2$ to 22. Simulation lasts for $T = 500$ time slots and γ is equal to 0.95. There is no minimum service ratio constraint. For this simulation we consider one more suboptimal algorithm, which is called *Proportional Fair Single Beam Allocation (PFSBA)* algorithm. This algorithm allocates only a single beam (and of course a single user) at each time slot. Simulation results reveal that taking the inter-beam interference into account significantly improves the logarithmic sum of average rates of users. An interesting result is that for $PFBA$ and $PFSBA$ algorithms log-sum rate first increases with K and then it starts to decrease. The reason is that increasing the number of users to a certain level decreases the individual rate, which decreases the log-sum rate.

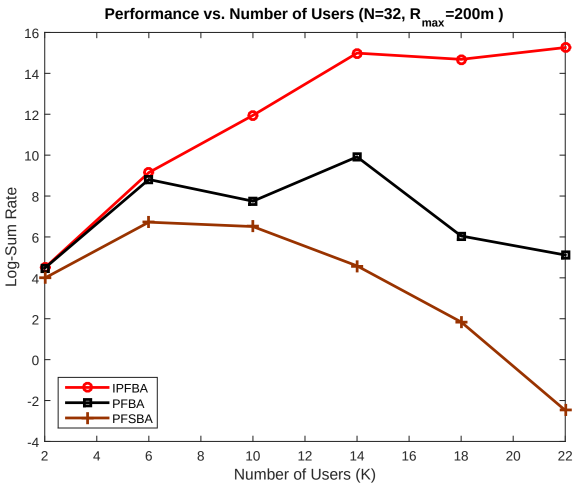

Fig. 4. Performance vs Number of Users for number of beams $N = 32$, maximum distance $R_{max} = 200$ meters and $\gamma = 0.95$.

Figure 5 shows the effect of increased number of beams for a fixed number of users. The results are interesting; for lower number of beams the performance of $IPFBA$ approaches to single beam allocation. The reason is that for low N, beamwidth is wider and sidelobe (hence inter-beam interference) is more significant. Therefore scheduling one beam (and user) per slot is a good choice. On the other hand, when the number of beams N, is significantly higher than users, beams are sharper and it is possible to schedule more users without causing significant interference. Hence, the performance of $PFBA$ approaches $IPFBA$.

VI. CONCLUSIONS

Simulation results show that the proposed proportional fair beam allocation algorithm performs almost optimally compared to MINLP-based solution. Moreover, instead of trying to increase the service ratio (number of user served at each time slot), fair resource allocation based on average received

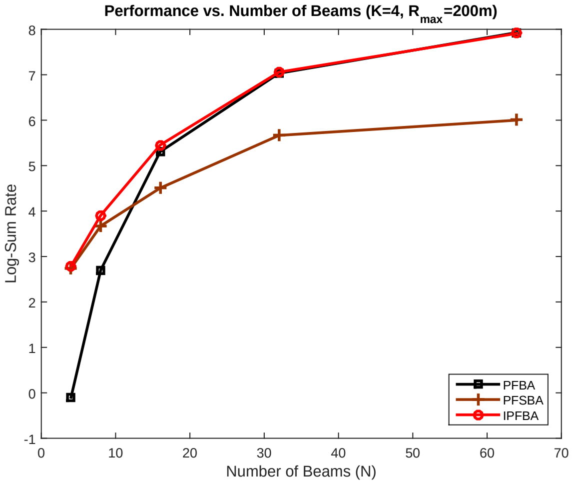

Fig. 5. Performance vs Number of Beams for number of users $K = 4$, maximum distance $R_{max} = 200$ meters and $\gamma = 0.95$.

rates and sidelobe interference performs much better. Finally, allocating a single beam and user at each time slot is optimal only for low number of beams and/or users, but it is largely suboptimal for large number of beams and users.

As future work, this research can further study realistic modulation coding schemes and develop a mixed integer linear program formulation as an improvement to the mixed integer nonlinear program formulation. MCS schemes and required received signal strengths defined for IEEE 802.11ad can be used in this direction.

Another direction of future work would be measuring the effects of sidelobe reduction on the scheduling schemes and their performance. There are recent works thet propose methods of sidelobe level reduction in switched-beam antenna arrays fed by Butler matrices [15]. Utilizing multiuser MIMO precoders is another alternative.

Proportional fairness (log-sum rate) metric is more suitable for networks with high bandwidth utilization and elastic traffic. In high bandwidth systems (such as mmWave), or in networks with inelastic traffic (such as VoIP) bandwidth utilization may be low. In such systems energy efficiency may be better performance metric than log-sum rate.

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