ELE 561 Kablosuz Haberleşme

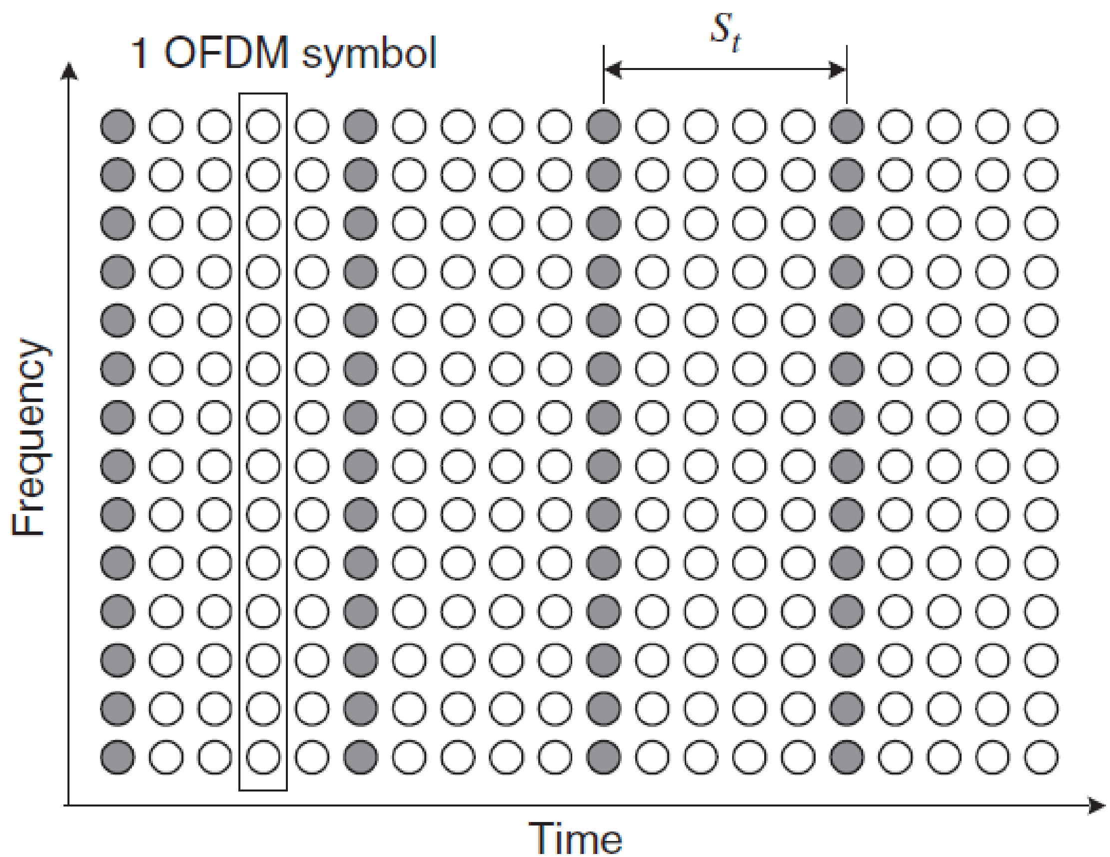
Konu 6: Kanal Kestirimi

6.1 Pilot Yapısı

- Alıcıda fft işleminden sonra 1-tap kanal denkleştirme yapılır.
- $X_l[k] = \frac{Y_l[k]}{H_l[k]}$
- Bu işin yapılabilmesi için $H_l[k]$'nın bilinmesi (kestirilmesi) gerekir.
- Pilot sinyaller kullanılır
    - Blok tipi
    - Tarak tipi
    - Kafes tipi

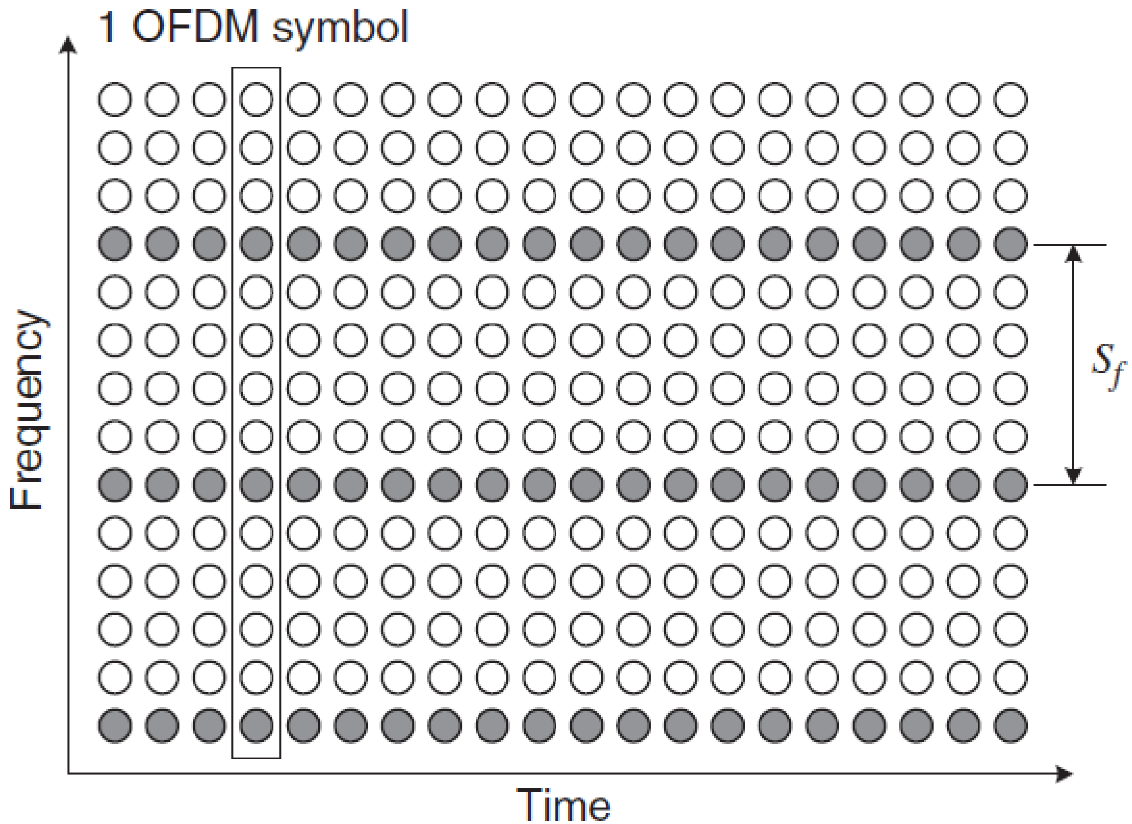
6.1.1 Blok tipi pilot sinyaller

- $S_t \leq \frac{1}{f_{Doppler}}$



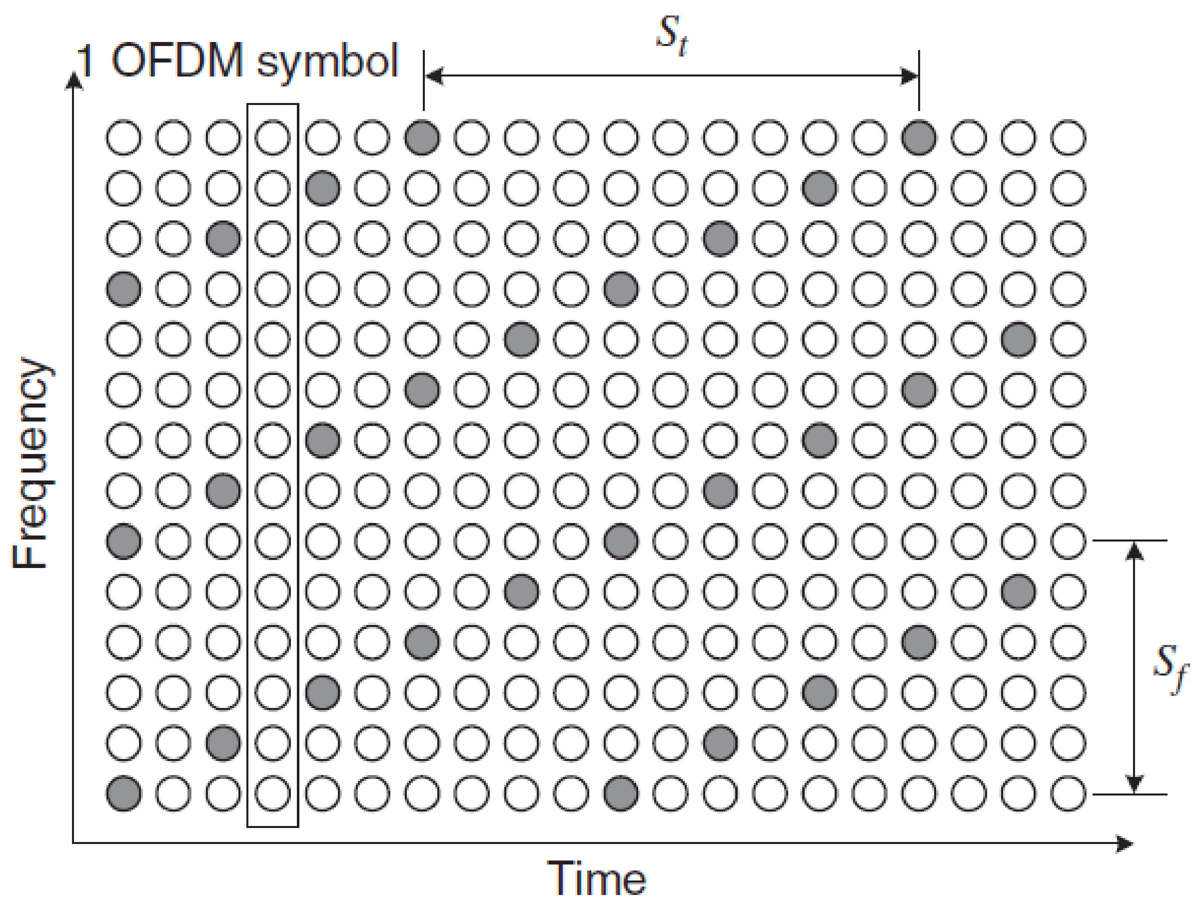
6.1.2 Tarak Tipi Pilot Sinyaller

- $S_f \leq \frac{1}{\sigma_d}$



6.1.3 Örgü Tipi Pilot Yapısı

- $S_t \leq \frac{1}{f_{Doppler}},\ S_f \leq \frac{1}{\sigma_d}$



6.2 Eğitim Dizisi ile Kanal Kestirimi

$$\mathbf{X} = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & X[N-1] \end{bmatrix}$$

$$\mathbf{Y} \triangleq \begin{bmatrix} Y[0] \\ Y[1] \\ \vdots \\ Y[N-1] \end{bmatrix} = \begin{bmatrix} X[0] & 0 & \cdots & 0 \\ 0 & X[1] & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & X[N-1] \end{bmatrix} \begin{bmatrix} H[0] \\ H[1] \\ \vdots \\ H[N-1] \end{bmatrix} + \begin{bmatrix} Z[0] \\ Z[1] \\ \vdots \\ Z[N-1] \end{bmatrix}$$

$$= \mathbf{XH} + \mathbf{Z}$$

6.2.1 En Küçük Kareler Kestirimi

$$
\begin{aligned}
J(\hat{\mathbf{H}}) &= \left\| \mathbf{Y} - \mathbf{X}\hat{\mathbf{H}} \right\|^2 \\
&= (\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}})^H (\mathbf{Y} - \mathbf{X}\hat{\mathbf{H}}) \\
&= \mathbf{Y}^H\mathbf{Y} - \mathbf{Y}^H\mathbf{X}\hat{\mathbf{H}} - \hat{\mathbf{H}}^H\mathbf{X}^H\mathbf{Y} + \hat{\mathbf{H}}^H\mathbf{X}^H\mathbf{X}\hat{\mathbf{H}}
\end{aligned} \tag{6.4}
$$

- $\frac{\partial J(\widehat{\boldsymbol{H}})}{\partial \widehat{\boldsymbol{H}}} = 0$

- $MSE_{LS} = E\{(\boldsymbol{H} - \boldsymbol{H}_{LS})^H (\boldsymbol{H} - \boldsymbol{H}_{LS})\}$

- Not: $\frac{\partial (X^T A X)}{\partial X} = 2X^T A, \frac{\partial AX}{\partial X} = A$

6.2.2 MMSE Kanal Kestirimi

- $\widehat{H} = WX^{-1}Y$
- $J(\widehat{H}) = E\left\{\left||e|\right|^2\right\} =$
- Diklik: Kestirim hatası kestirilen şeye dik olmalı

$$\begin{aligned} E\{\mathbf{e}\tilde{\mathbf{H}}^H\} &= E\{(\mathbf{H}-\hat{\mathbf{H}})\tilde{\mathbf{H}}^H\} \\ &= E\{(\mathbf{H}-\mathbf{W}\tilde{\mathbf{H}})\tilde{\mathbf{H}}^H\} \\ &= E\{\mathbf{H}\tilde{\mathbf{H}}^H\}-\mathbf{W}E\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\} \\ &= \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}-\mathbf{W}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}} = \mathbf{0} \end{aligned} \tag{6.10}$$

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}} &= E\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\} \\ &= E\{\mathbf{X}^{-1}\mathbf{Y}(\mathbf{X}^{-1}\mathbf{Y})^H\} \\ &= E\{(\mathbf{H}+\mathbf{X}^{-1}\mathbf{Z})(\mathbf{H}+\mathbf{X}^{-1}\mathbf{Z})^H\} \\ &= E\{\mathbf{H}\mathbf{H}^H+\mathbf{X}^{-1}\mathbf{Z}\mathbf{H}^H+\mathbf{H}\mathbf{Z}^H(\mathbf{X}^{-1})^H+\mathbf{X}^{-1}\mathbf{Z}\mathbf{Z}^H(\mathbf{X}^{-1})^H)\} \\ &= E\{\mathbf{H}\mathbf{H}^H\}+E\{\mathbf{X}^{-1}\mathbf{Z}\mathbf{Z}^H(\mathbf{X}^{-1})^H\} \\ &= E\{\mathbf{H}\mathbf{H}^H\}+\frac{\sigma_z^2}{\sigma_x^2}\mathbf{I} \end{aligned} \tag{6.13}$$

6.2.2 MMSE Kestirimi

$$\hat{\mathbf{H}} = \mathbf{W}\tilde{\mathbf{H}} = \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}\mathbf{R}_{\tilde{\mathbf{H}}\tilde{\mathbf{H}}}^{-1}\tilde{\mathbf{H}}$$

$$= \mathbf{R}_{\mathbf{H}\tilde{\mathbf{H}}}\left(\mathbf{R}_{\mathbf{H}\mathbf{H}} + \frac{\sigma_z^2}{\sigma_x^2}\mathbf{I}\right)^{-1}\tilde{\mathbf{H}} \quad (6.14)$$

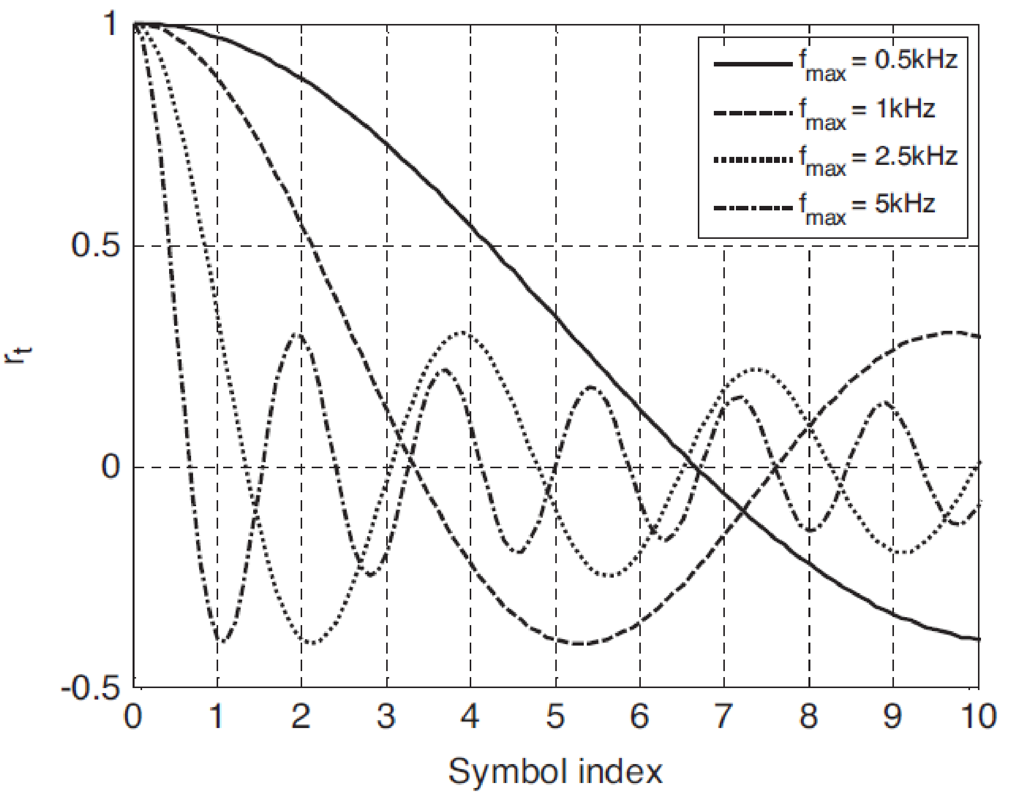
$$E\left\{h_{k,l}\tilde{h}^*_{k',l'}\right\} = E\left\{h_{k,l}h^*_{k',l'}\right\} = r_f[k-k']r_t[l-l'] \quad (6.15)$$

$$r_f[k] = \frac{1}{1 + j2\pi\tau_{rms}k\Delta f} \quad (6.16)$$

$$r_t[l] = J_0(2\pi f_{\max} l T_{sym}) \quad (6.17)$$

6.2.2 MMSE Kestirimi

- Pilot sembollerin kanalları kestirildikten sonra interpolasyon yapılır
  - linear interpolation,
  - second-order polynomial interpolation,
  - cubic spline interpolation



6.3 DFT Tabanlı Kanal Kestirimi

- Kanal kestirilir $\widehat{H}[k]$ bulunur (N uzunluklu).
- IDFT alınır, yine N uzunluklu impulse response elde edilir.
  - bunun kanal uzunluğu kadarı alınır. Impulse response'un L uzunluklu olduğunu biliyoruz. Geri kalan N-L 'lik kısım sıfıra eşitlenir.
- Tekrar DFT alınır.

$$\text{IDFT}\{\hat{H}[k]\} = h[n] + z[n] \triangleq \hat{h}[n], \quad n = 0, 1, \ldots, N-1 \tag{6.18}$$

$$\hat{h}_{DFT}[n] = \begin{cases} h[n] + z[n], & n = 0, 1, 2, \cdots, L-1 \\ 0, & \text{otherwise} \end{cases} \tag{6.19}$$

$$\hat{H}_{\text{DFT}}[k] = \text{DFT}\left\{\hat{h}_{\text{DFT}}(n)\right\} \tag{6.20}$$

6.3 DFT Tabanlı Kestirim

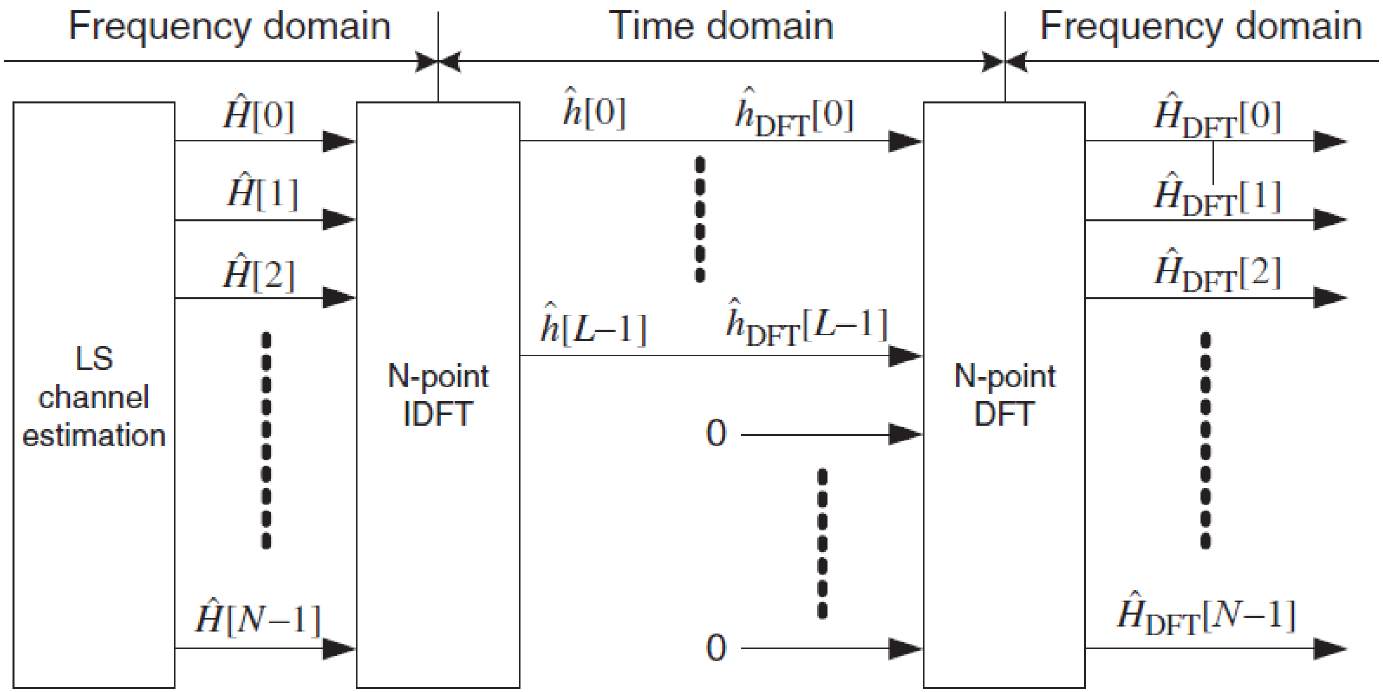
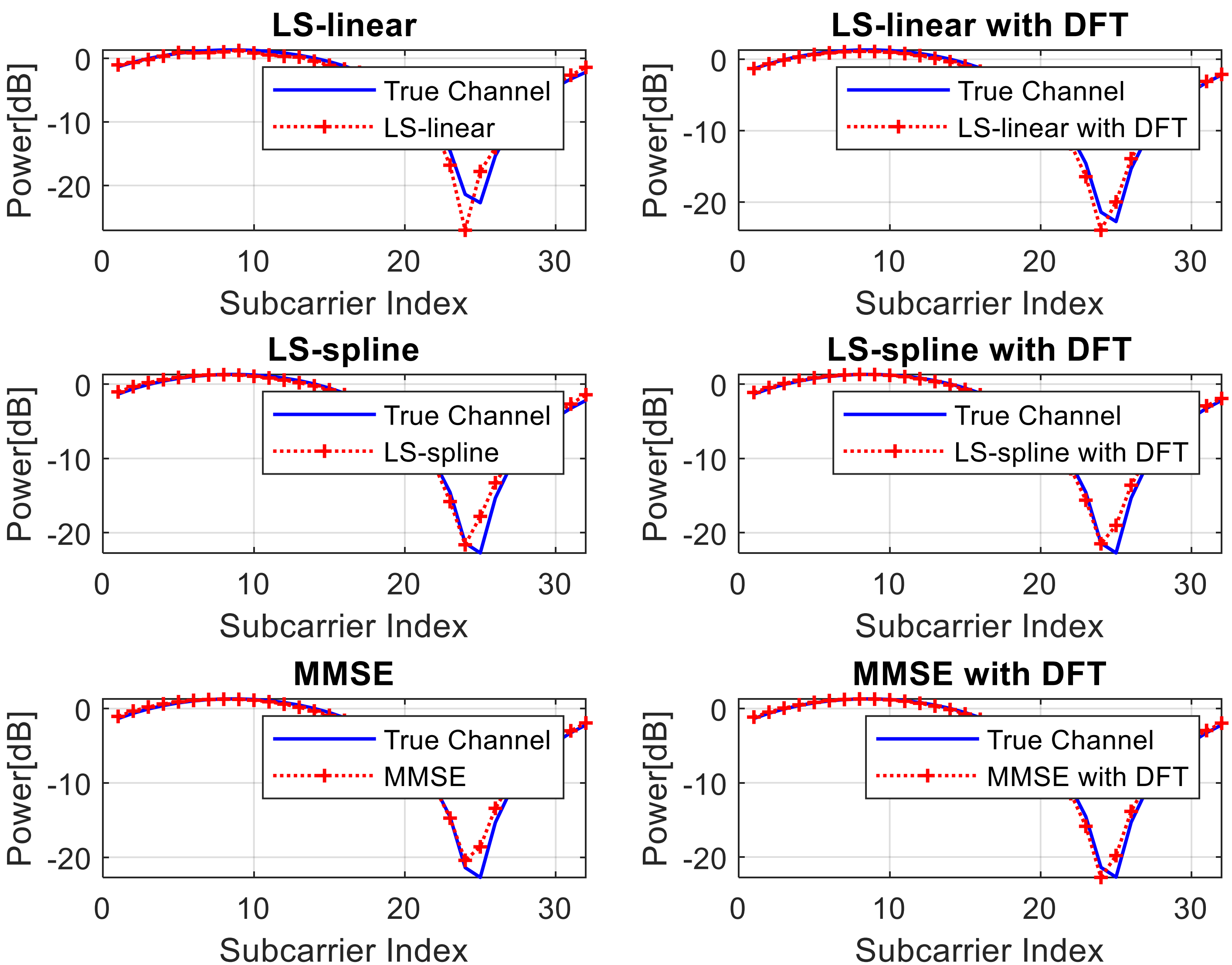


Figure 6.6 DFT-based channel estimation.

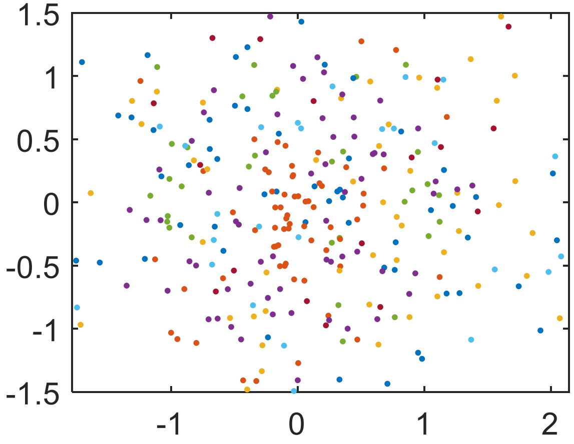
Programlar

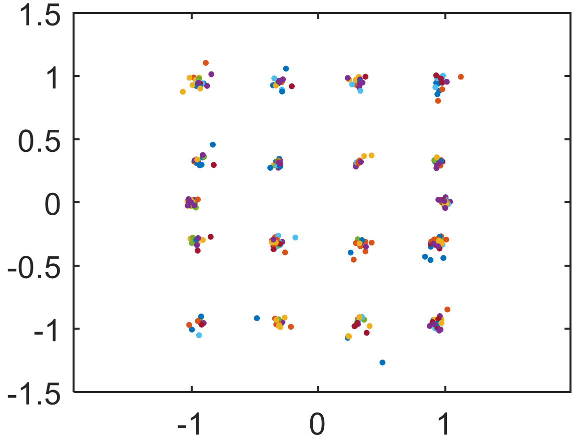
- Program 6.1 “LS_CE” for LS channel estimation method
- Program 6.2 “MMSE_CE” for MMSE channel estimation method
- Program 6.3 “interpolate” for channel interpolation between pilots
- Program 6.4 “channel_estimation.m” for DFT-based channel estimation

SNR=30

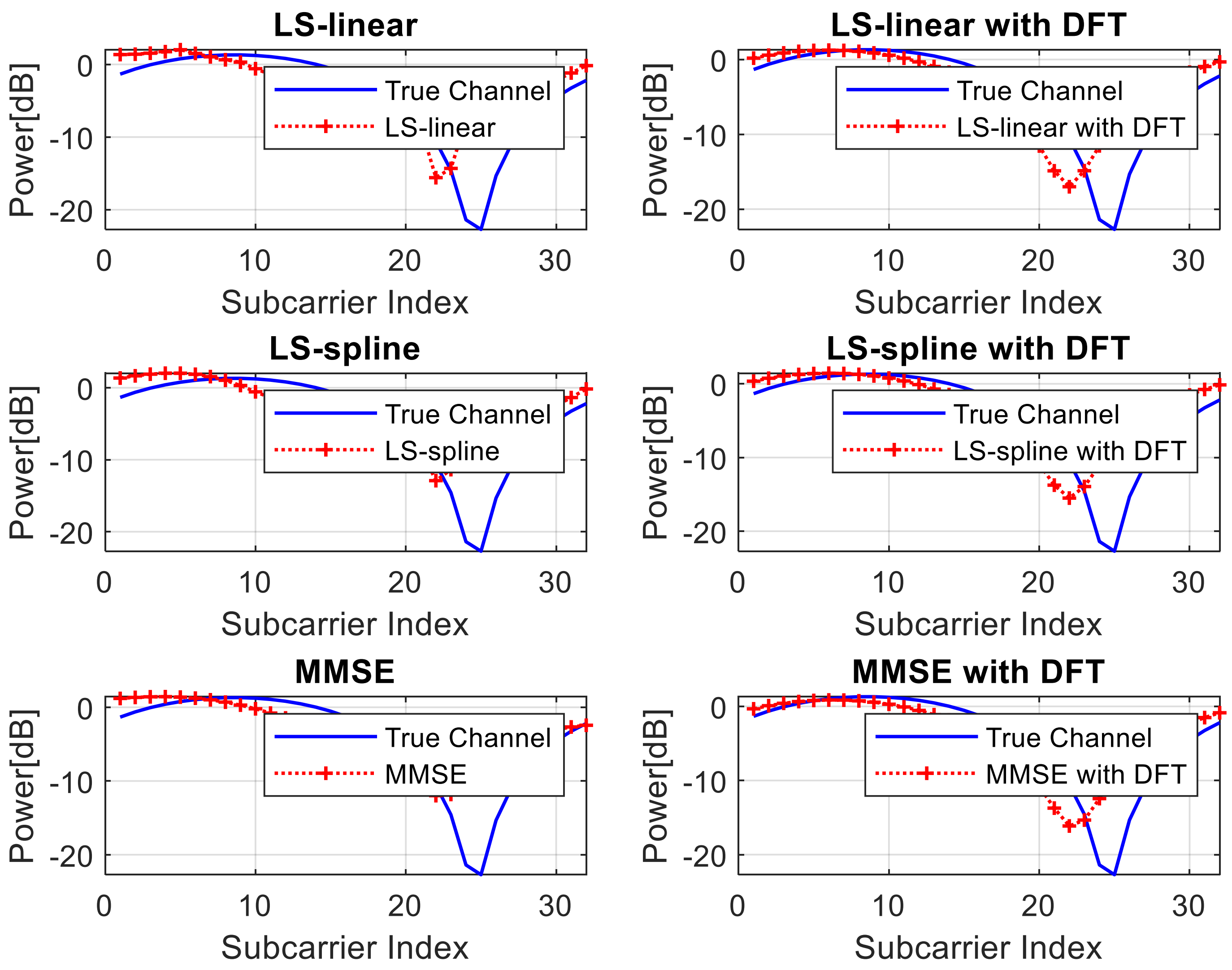

LS-linear
True Channel
LS-linear
Power[dB]
Subcarrier Index
LS-linear with DFT
True Channel
LS-linear with DFT
LS-spline
True Channel
LS-spline
LS-spline with DFT
True Channel
LS-spline with DFT
MMSE
True Channel
MMSE
MMSE with DFT
True Channel
MMSE with DFT

SNR=30



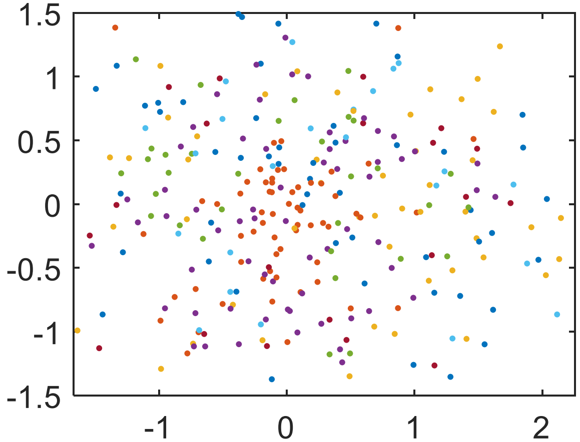


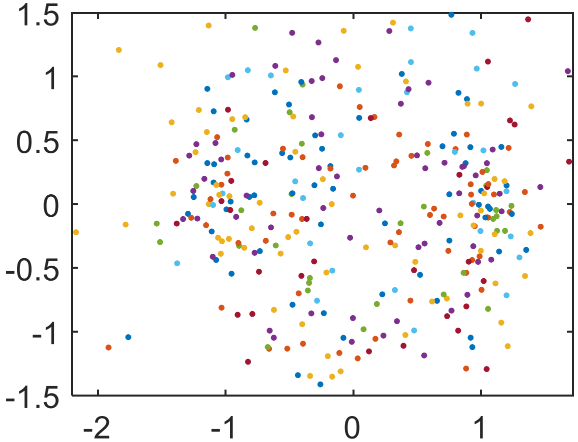
SNR=10



LS-linear
True Channel
LS-linear
LS-linear with DFT
True Channel
LS-linear with DFT
LS-spline
True Channel
LS-spline
LS-spline with DFT
True Channel
LS-spline with DFT
MMSE
True Channel
MMSE
MMSE with DFT
True Channel
MMSE with DFT
Power[dB]
Subcarrier Index
0
-10
-20
0
10
20
30

SNR=10





Diğer konular

- 6.4 Decision Directed Channel Estimation
  - İlk başta pilotlar kullanılarak kestirim yapılır, sonra veri sembolleriyle bu tahmin güncellenir.
- 6.5 Advanced Channel Estimation techniques
  - 6.5.1 Channel Estimation Using a Superimposed Signal
  - 6.5.2 Channel Estimation in Fast Time-Varying Channels
  - 6.5.3 EM Algorithm-Based Channel Estimation
    - Expectation-Maximization
  - 6.5.4 Blind Channel Estimation
    - Pilot kullanmadan kestirim
- LS veya MMSE bizim için yeterli