MIMO-OFDM Wireless Communications with MATLAB®

SISO : $C = W \log_2 \left(1 + \frac{P|h|^2}{\sigma^2}\right)$

$\min(N_R, N_T)$ çarpan (MIMO)

Chapter 9. MIMO: Kanal Kapasitesi

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Chapter 9. MIMO: Kanal Kapasitesi

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$\mathbf{H}_{N_R \times N_T}$ matris

rate factor $\min(N_R, N_T)$

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Chapter 9. MIMO: Kanal Kapasitesi

9.1 Gerekli Matris Teorisi

singular value decomposition (svd)

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \tag{9.1}$$

$H_{N_R \times N_T}$

$U_{N_R \times N_R}$ unitary: $UU^H = I$

singular value → $\Sigma_{N_R \times N_T}$: diag$(\sigma_1, \sigma_2,$

$V = N_T \times N_T$ unitary: $VV^H = I$

$$\begin{aligned}\mathbf{H} &= \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \\ &= \underbrace{[\mathbf{U}_{N_{\min}}\ \mathbf{U}_{N_R-N_{\min}}]}_{\mathbf{U}} \underbrace{\begin{bmatrix}\boldsymbol{\Sigma}_{N_{\min}} \\ \mathbf{0}_{N_R-N_{\min}}\end{bmatrix}}_{\boldsymbol{\Sigma}} \mathbf{V}^H \\ &= \mathbf{U}_{N_{\min}}\boldsymbol{\Sigma}_{N_{\min}}\mathbf{V}^H\end{aligned} \tag{9.2}$$

$$\begin{aligned}\mathbf{H} &= \mathbf{U}\underbrace{[\boldsymbol{\Sigma}_{N_{\min}}\ \mathbf{0}_{N_T-N_{\min}}]}_{\boldsymbol{\Sigma}} \underbrace{\begin{bmatrix}\mathbf{V}^H_{N_{\min}} \\ \mathbf{V}^H_{N_T-N_{\min}}\end{bmatrix}}_{\mathbf{V}^H} \\ &= \mathbf{U}\boldsymbol{\Sigma}_{N_{\min}}\mathbf{V}^H_{N_{\min}}\end{aligned} \tag{9.3}$$

9.1 Gerekli Matris Teorisi

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{\Sigma}\mathbf{\Sigma}^H\mathbf{U}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H \tag{9.4}$$

$$\lambda_i = \begin{cases} \sigma_i^2, & \text{if } i = 1, 2, \cdots, N_{\min} \\ 0, & \text{if } i = N_{\min}+1, \cdots, N_R. \end{cases} \tag{9.5}$$

$$\mathbf{H}\underbrace{[\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_n]}_{\mathbf{X}} = \underbrace{[\mathbf{x}_1\ \mathbf{x}_2 \cdots\ \mathbf{x}_n]}_{\mathbf{X}}\mathbf{\Lambda}_{\text{non-}H} \tag{9.6}$$

9.1 Gerekli Matris Teorisi

$$\mathbf{H} = \mathbf{X}\mathbf{\Lambda}_{\text{non-}H}\mathbf{X}^{-1} \tag{9.7}$$

$$\|\mathbf{H}\|_F^2 = \mathrm{Tr}(\mathbf{H}\mathbf{H}^H) = \sum_{i=1}^{N_R}\sum_{j=1}^{N_T}|h_{i,j}|^2. \tag{9.8}$$

$$\begin{aligned}\|\mathbf{H}\|_F^2 &= \|\mathbf{Q}^H\mathbf{H}\|_F^2 \\ &= \mathrm{Tr}(\mathbf{Q}^H\mathbf{H}\mathbf{H}^H\mathbf{Q}) \\ &= \mathrm{Tr}(\mathbf{Q}^H\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H\mathbf{Q}) \\ &= \mathrm{Tr}(\mathbf{\Lambda}) \\ &= \sum_{i=1}^{N_{\min}}\lambda_i \\ &= \sum_{i=1}^{N_{\min}}\sigma_i^2\end{aligned} \tag{9.9}$$

9.2 Deterministik MIMO Kanal kapasitesi

$$\mathbf{y} = \sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{H}\mathbf{x} + \mathbf{z} \qquad (9.10)$$

$$\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^H\}. \qquad (9.11)$$

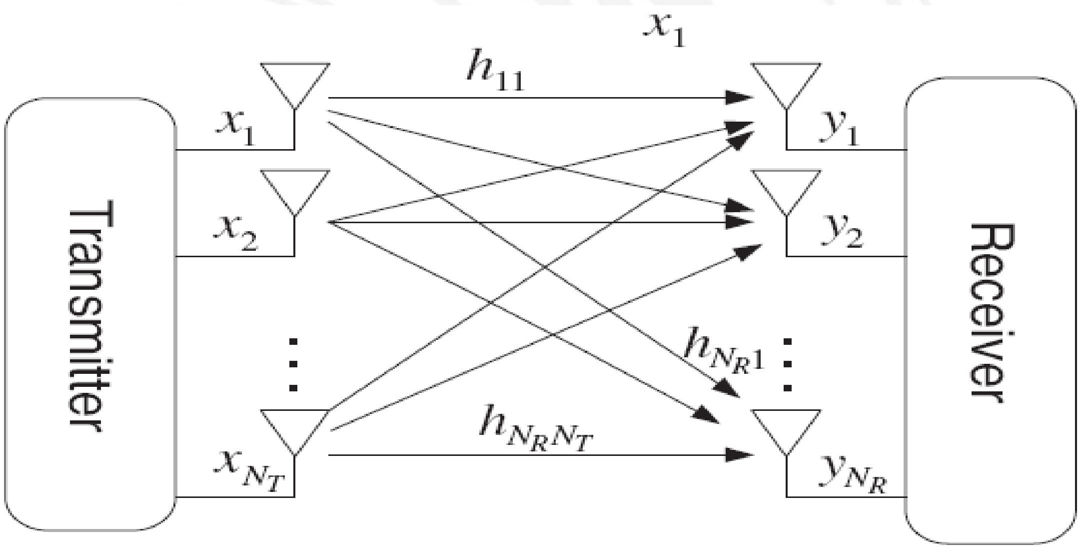

Figure 9.1 $N_R \times N_T$ MIMO system.

9.2.1 Vericide kanalın bilindiği durum

$$C = \max_{f(\mathbf{x})} I(\mathbf{x};\mathbf{y}) \text{ bits/channel use} \tag{9.12}$$

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}) \tag{9.13}$$

$$H(\mathbf{y}|\mathbf{x}) = H(\mathbf{z}) \tag{9.14}$$

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{z}) \tag{9.15}$$

9.2.1 Vericide kanalın bilindiği durum

$$\begin{aligned}\mathbf{R}_{yy} &= E\{\mathbf{y}\mathbf{y}^H\} = E\left\{\left(\sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{H}\mathbf{x}+\mathbf{z}\right)\left(\sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{x}^H\mathbf{H}^H+\mathbf{z}^H\right)\right\}\\ &= E\left\{\left(\frac{\mathsf{E}_x}{N_T}\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H+\mathbf{z}\mathbf{z}^H\right)\right\}\\ &= \frac{\mathsf{E}_x}{N_T}E\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H+\mathbf{z}\mathbf{z}^H\}\\ &= \frac{\mathsf{E}_x}{N_T}\mathbf{H}E\{\mathbf{x}\mathbf{x}^H\}\mathbf{H}^H+E\{\mathbf{z}\mathbf{z}^H\}\\ &= \frac{\mathsf{E}_x}{N_T}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H+\mathsf{N}_0\mathbf{I}_{N_R}\end{aligned} \tag{9.16}$$

$$\begin{aligned}H(\mathbf{y}) &= \log_2\{\det(\pi e\mathbf{R}_{yy})\}\\ H(\mathbf{z}) &= \log_2\{\det(\pi e\mathsf{N}_0\mathbf{I}_{N_R})\}\end{aligned} \tag{9.17}$$

9.2.1 Vericide kanalın bilindiği durum

$$I(\mathbf{x};\mathbf{y}) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{ bps/Hz.} \tag{9.18}$$

$$C = \max_{Tr(\mathbf{R}_{xx})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H\right) \text{ bps/Hz.} \tag{9.19}$$

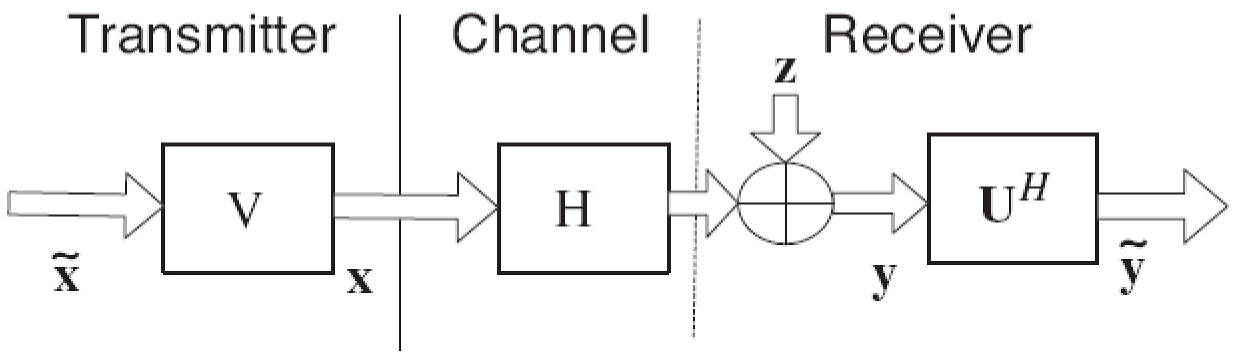

Figure 9.2 Modal decomposition when CSI is available at the transmitter side.

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9.2.1 Vericide kanalın bilindiği durum (Precoding ve Post-processing)

$$\tilde{\mathbf{y}} = \sqrt{\frac{\mathsf{E}_x}{N_T}}\mathbf{U}^H\mathbf{H}\mathbf{V}\tilde{\mathbf{x}} + \tilde{\mathbf{z}} \quad (9.20)$$

$$\tilde{y}_i = \sqrt{\frac{\mathsf{E}_x}{N_T}}\sqrt{\lambda_i}\tilde{x}_i + \tilde{z}_i, \quad i = 1, 2, \cdots, r. \quad (9.21)$$

$$C_i(\gamma_i) = \log_2\left(1 + \frac{\mathsf{E}_x\gamma_i}{N_T\mathsf{N}_0}\lambda_i\right), \quad i = 1, 2, \cdots, r. \quad (9.22)$$

$$E\{\mathbf{x}^H\mathbf{x}\} = \sum_{i=1}^{N_T} E\left\{|x_i|^2\right\} = N_T. \quad (9.23)$$

9.2.1 Vericide kanalın bilindiği durum (Uzamsal sanal paralel kanallar)

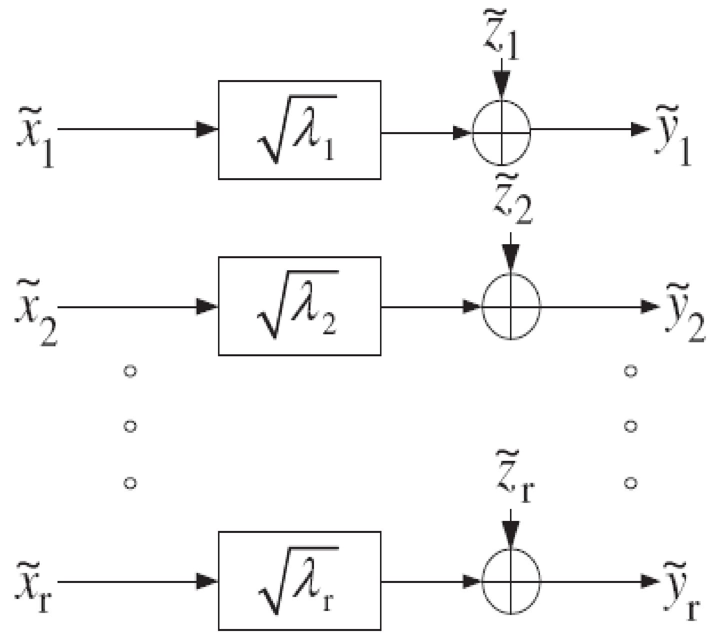

Figure 9.3 The *r* virtual SISO channels obtained from the modal decomposition of a MIMO channel.

9.2.1 Vericide kanalın bilindiği durum (Su doldurma)

$$C = \sum_{i=1}^{r} C_i(\gamma_i) = \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathrm{E_x}\gamma_i}{N_T \mathrm{N_0}}\lambda_i\right) \quad (9.24)$$

$$C = \max_{\{\gamma_i\}} \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathrm{E_x}\gamma_i}{N_T \mathrm{N_0}}\lambda_i\right) \quad (9.25)$$

$$\gamma_i^{opt} = \left(\mu - \frac{N_T \mathrm{N_0}}{\mathrm{E_x}\lambda_i}\right)^+, \quad i = 1, \cdots, r \quad (9.26)$$

$$\sum_{i=1}^{r} \gamma_i^{opt} = N_T. \quad (9.27)$$

$$(x)^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}. \quad (9.28)$$

9.2.1 Vericide kanalın bilindiği durum (Su doldurma görseli)

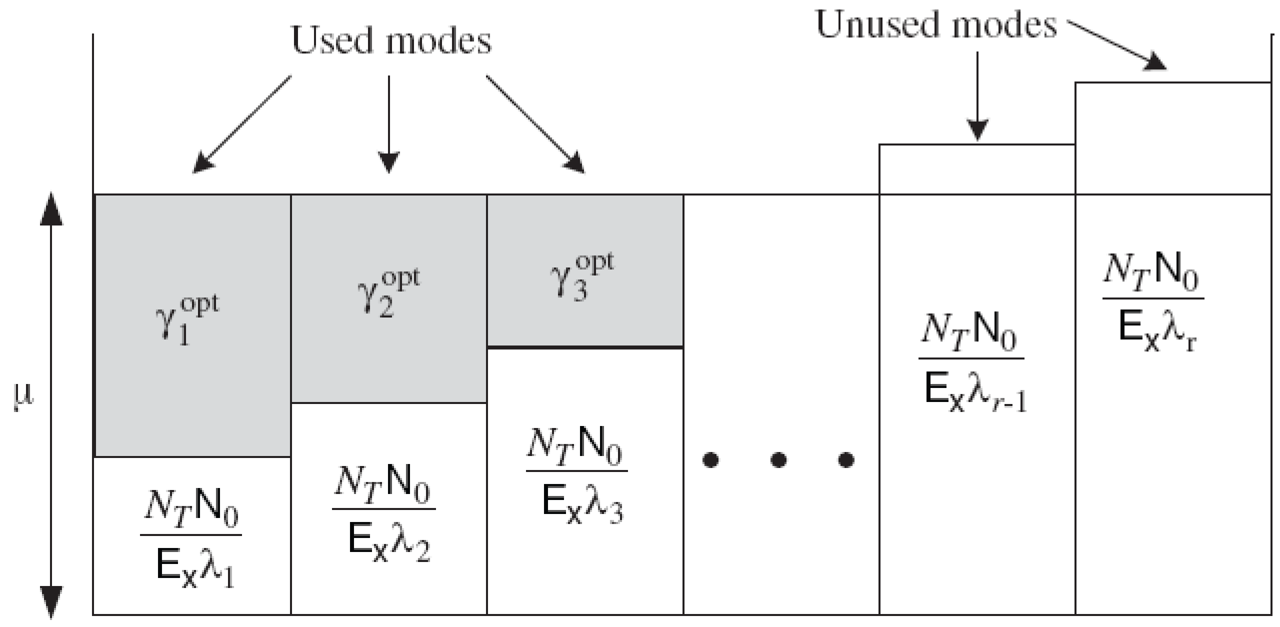

Figure 9.4 Water-pouring power allocation algorithm.

9.2.2 Vericide kanalın bilinmediği durum

$$\mathbf{R}_{xx} = \mathbf{I}_{N_T} \tag{9.29}$$

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_\mathrm{x}}{N_T \mathsf{N}_0}\mathbf{H}\mathbf{H}^H\right). \tag{9.30}$$

$$\begin{aligned} C &= \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_\mathrm{x}}{N_T \mathsf{N}_0}\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H\right) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_\mathrm{x}}{N_T \mathsf{N}_0}\boldsymbol{\Lambda}\right) \\ &= \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathsf{E}_\mathrm{x}}{N_T \mathsf{N}_0}\lambda_i\right) \end{aligned} \tag{9.31}$$

9.2.2 Vericide kanalın bilinmediği durum

$$\lambda_i = \frac{\zeta}{N}, \quad i = 1, 2, \cdots, N. \tag{9.32}$$

$$\mathbf{H}\mathbf{H}^H = \mathbf{H}^H\mathbf{H} = \frac{\zeta}{N}\mathbf{I}_N \tag{9.33}$$

$$C = N \log_2\left(1 + \frac{\zeta \mathsf{E}_{\mathsf{x}}}{\mathsf{N}_0 N}\right). \tag{9.34}$$

9.2.3 SIMO ve MISO Kanal Kapasitesi (MIMO'nun özel durumu)

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}\|\mathbf{h}\|_F^2\right). \qquad (9.35)$$

$$C_{SIMO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}N_R\right). \qquad (9.36)$$

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\|\mathbf{h}\|_F^2\right). \qquad (9.37)$$

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}\right). \qquad (9.38)$$

9.2.3 SIMO ve MISO Kanal Kapasitesi (MIMO'nun özel durumu)

$$y = \sqrt{\mathsf{E}_x}\mathbf{h} \cdot \frac{\mathbf{h}^H}{\|\mathbf{h}\|} x + z = \sqrt{\mathsf{E}_x}\|\mathbf{h}\| x + z \qquad (9.39)$$

$$C_{MISO} = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0}\|\mathbf{h}\|_F^2\right) = \log_2\left(1 + \frac{\mathsf{E}_x}{\mathsf{N}_0} N_T\right). \qquad (9.40)$$

9.3 Rastgele MIMO Kanal Kapasitesi (Beklenen değer ve Kesinti Olasılığı)

$$\overline{C} = E\{C(\mathbf{H})\} = E\left\{ \max_{\mathrm{Tr}(\mathbf{R}_{xx})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E_x}}{N_T \mathsf{N_0}} \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H \right) \right\} \qquad (9.41)$$

$$\overline{C_{OL}} = E\left\{ \sum_{i=1}^{r} \log_2 \left(1 + \frac{\mathsf{E_x}}{N_T \mathsf{N_0}} \lambda_i \right) \right\}. \qquad (9.42)$$

$$\overline{C}_{CL} = E\left\{ \max_{\sum_{i=1}^{r} \gamma_i = N_T} \sum_{i=1}^{r} \log_2 \left(1 + \frac{\mathsf{E_x}}{N_T \mathsf{N_0}} \gamma_i \lambda_i \right) \right\} \qquad (9.43)$$

$$= E\left\{ \sum_{i=1}^{r} \log_2 \left(1 + \frac{\mathsf{E_x}}{N_T \mathsf{N_0}} \gamma_i^{opt} \lambda_i \right) \right\}. \qquad (9.44)$$

$$P_{out}(R) = \Pr(C(\mathbf{H}) < R) \qquad (9.45)$$

Kodlar

- Program 9.1 “Ergodic_Capacity_CDF.m” for ergodic capacity of MIMO channel
- Program 9.2 “Ergodic_Capacity_vs_SNR.m” for ergodic channel capacity vs. SNR in Figure 9.6.
- Program 9.3 “OL_CL_Comparison.m” for Ergodic channel capacity: open-loop vs. closedloop
- Program 9.4 “Water_Pouring” for water-pouring algorithm
- Program 9.5 “Ergodic_Capacity_Correlation.m:” Channel capacity reduction due to correlation

9.3 Rastgele MIMO Kanal Kapasitesi: CDF

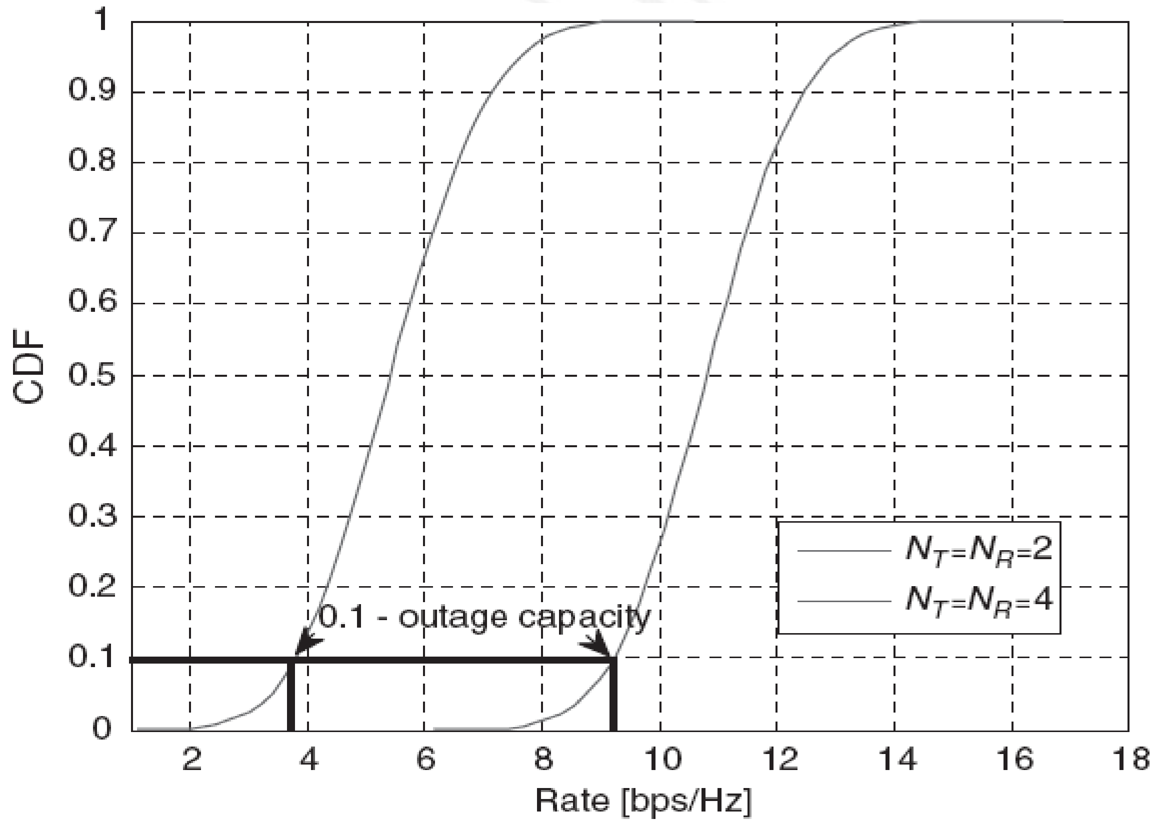

Figure 9.5 Distribution of MIMO channel capacity (SNR = 10dB; CSI is not available at the transmitter side).

9.3 Rastgele MIMO Kanal Kapasitesi: vs SNR

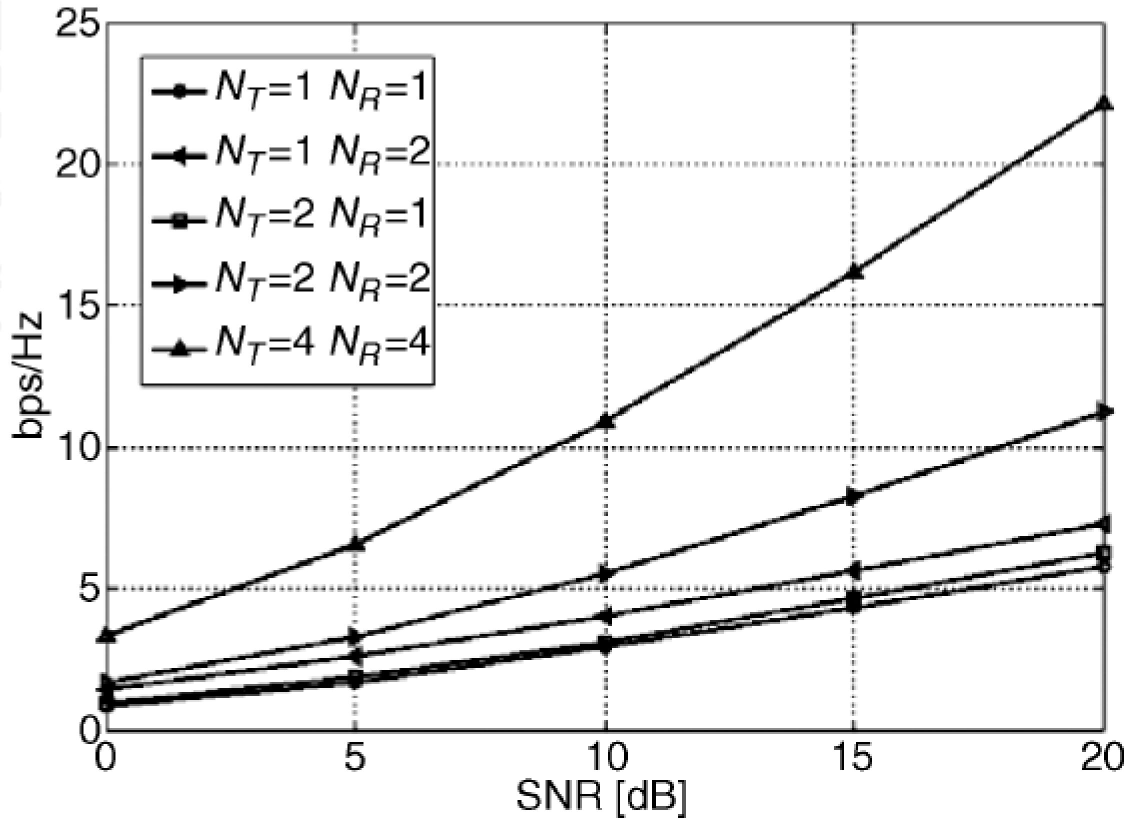

Figure 9.6 Ergodic MIMO channel capacity when CSI is not available at the transmitter.

9.3 Rastgele MIMO Kanal Kapasitesi

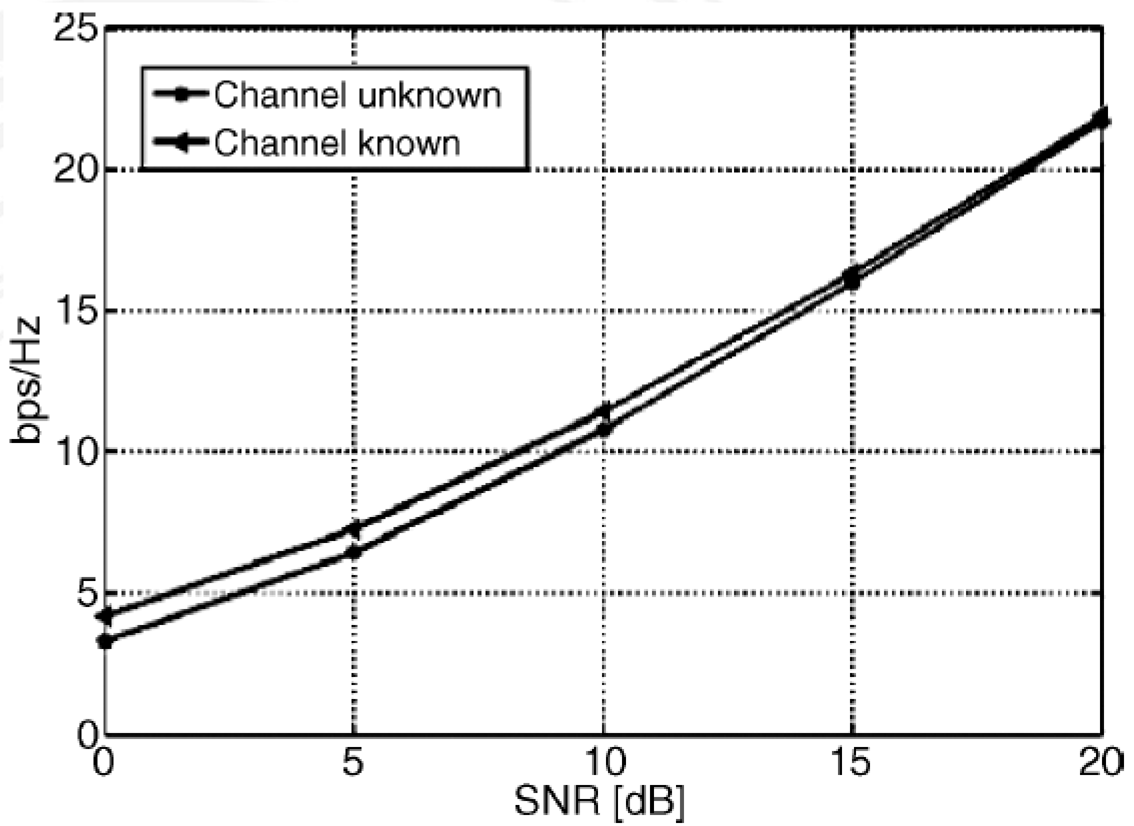

Figure 9.7 Ergodic channel capacity: $N_T = N_R = 4$.

9.3 Rastgele MIMO Kanal Kapasitesi: Antenler arası korelasyon var ise

$$C \approx \max_{\mathrm{Tr}(\mathbf{R}_{xx})=N} \log_2 \det(\mathbf{R}_{xx}) + \log_2 \det\left(\frac{\mathsf{E}_x}{N\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right) \tag{9.46}$$

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{9.47}$$

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t\mathbf{H}_w^H\mathbf{R}_r^{H/2}\right). \tag{9.48}$$

$$C \approx \log_2 \det\left(\frac{\mathsf{E}_x}{N_T\mathsf{N}_0}\mathbf{H}_w\mathbf{H}_w^H\right) + \log_2 \det(\mathbf{R}_r) + \log_2 \det(\mathbf{R}_t). \tag{9.49}$$

9.3 Rastgele MIMO Kanal Kapasitesi Antenler arası korelasyon var ise

$$\log_2 \det(\mathbf{R}_r) + \log_2 \det(\mathbf{R}_t). \quad (9.50)$$

$$\det(\mathbf{R}) = \prod_{i=1}^{N} \lambda_i. \quad (9.51)$$

$$\left(\Pi_{i=1}^{N} \lambda_i\right)^{\frac{1}{N}} \leq \frac{1}{N} \sum_{i=1}^{N} \lambda_i = 1. \quad (9.52)$$

$$\log_2 \det(\mathbf{R}) \leq 0 \quad (9.53)$$

9.3 Rastgele MIMO Kanal Kapasitesi
Antenler arası korelasyon var ise

$$\mathbf{R}_t = \begin{bmatrix} 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} & 0.25e^{j0.53\pi} \\ 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} & 0.43e^{j0.35\pi} \\ 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 & 0.76e^{j0.17\pi} \\ 0.25e^{-j0.53\pi} & 0.43e^{-j0.35\pi} & 0.76e^{-j0.17\pi} & 1 \end{bmatrix} \quad (9.54)$$

9.3 Rastgele MIMO Kanal Kapasitesi
Antenler arası korelasyon var ise

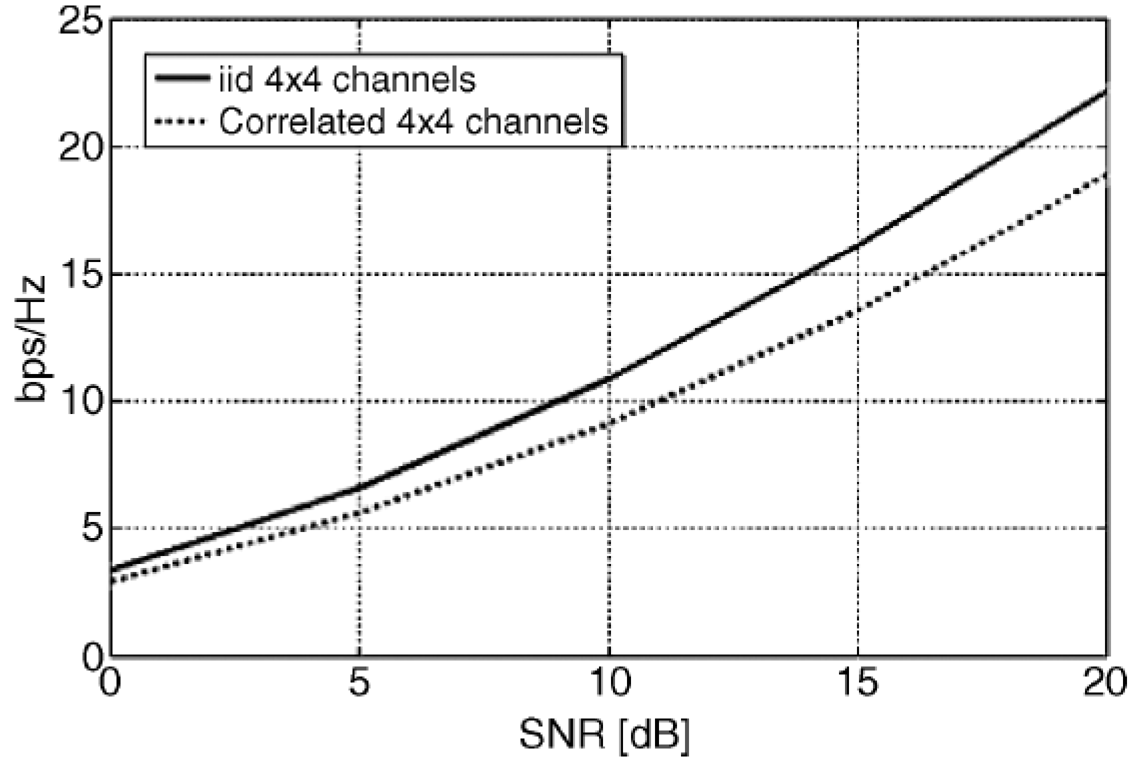

Figure 9.8 Capacity reduction due to the channel correlation.