

Chapter 10: Antenna Diversity and Space –Time Coding Techniques

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10.1 Antenna Diversity

- İki ana yöntem

- Çeşitleme (Diversity) : Aynı bilginin farklı kanallardan gönderilmesi;
- Çoklama (Multiplexing) Her kanaldan farklı bilgi gönderilmesi;

- Anten Çeşitlemesi

- Sönümlü kanalın değişkenliğine karşı hata performansının kötüleşmesini önler
- Uzamsal Çeşitleme (Space) Çoklu anten
- Polarizasyon çeşitlemesi
- Zaman çeşitlemesi
- Frekans çeşitlemesi
- Açı çeşitlemesi

Çeşitleme türleri

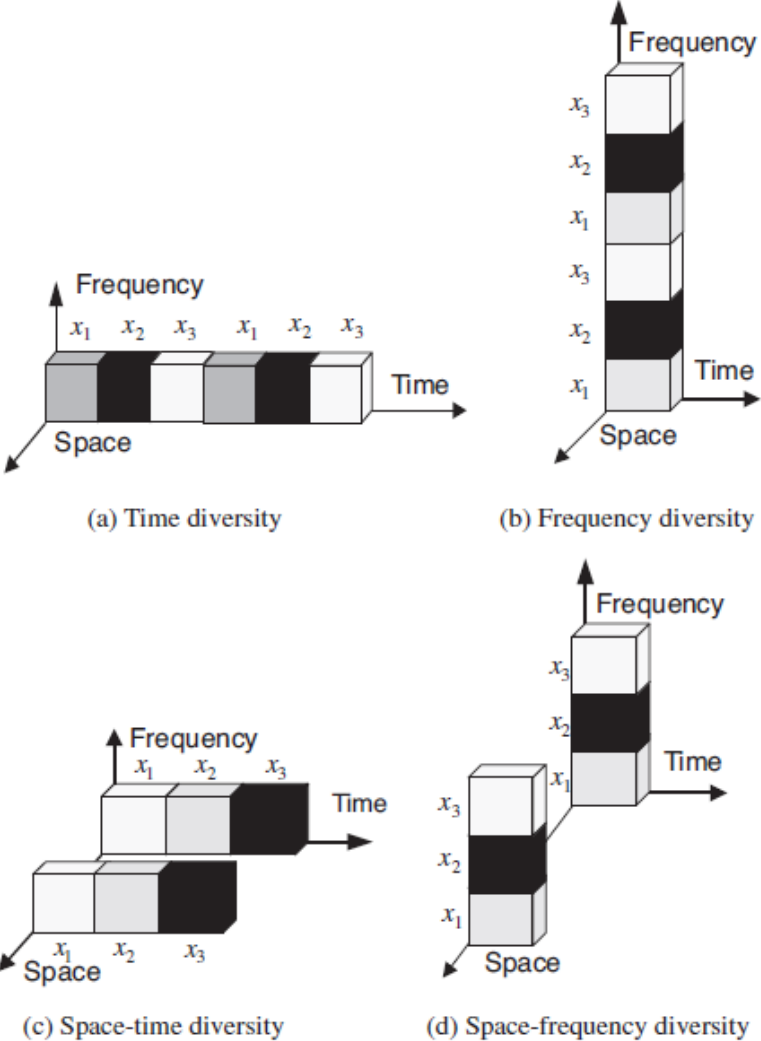


Figure 10.1 Illustration of time, frequency, and space diversity techniques.

Anten konfigürasyonları

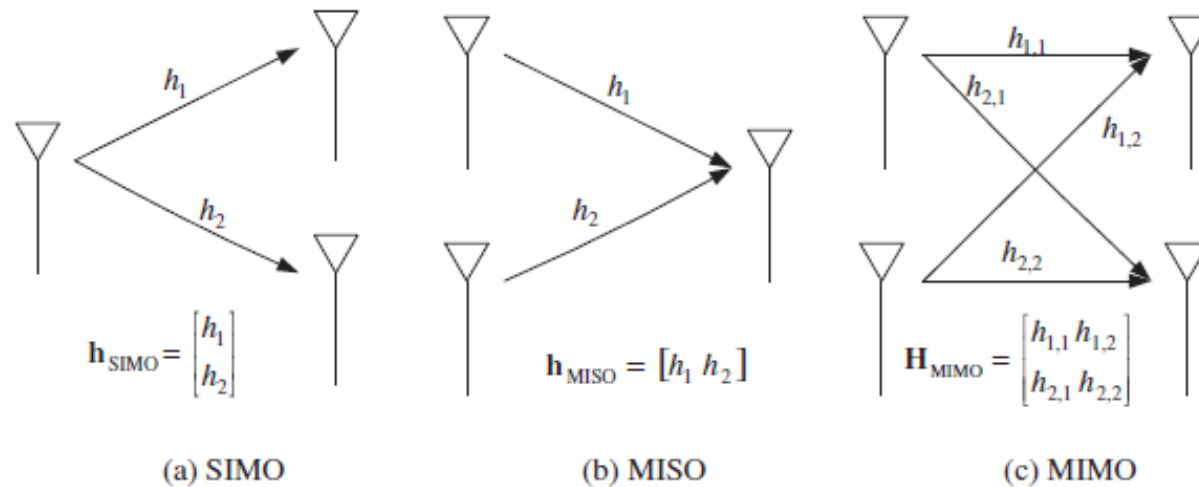


Figure 10.2 Examples of various antenna configurations.

10.1.1. Alıcıda Çeşitleme

$$\mathbf{h} = [h_1 h_2 \cdots h_{N_R}]^T \quad (10.1)$$

$$\mathbf{y} = \sqrt{\frac{E_x}{N_0}} \mathbf{h}x + \mathbf{z} \quad (10.2)$$

- Selection (SC)

$$\gamma_i = |h_i|^2 \frac{E_x}{N_0}, \quad i = 1, 2, \dots, N_R. \quad (10.3)$$

- Maximal Ratio (MC)

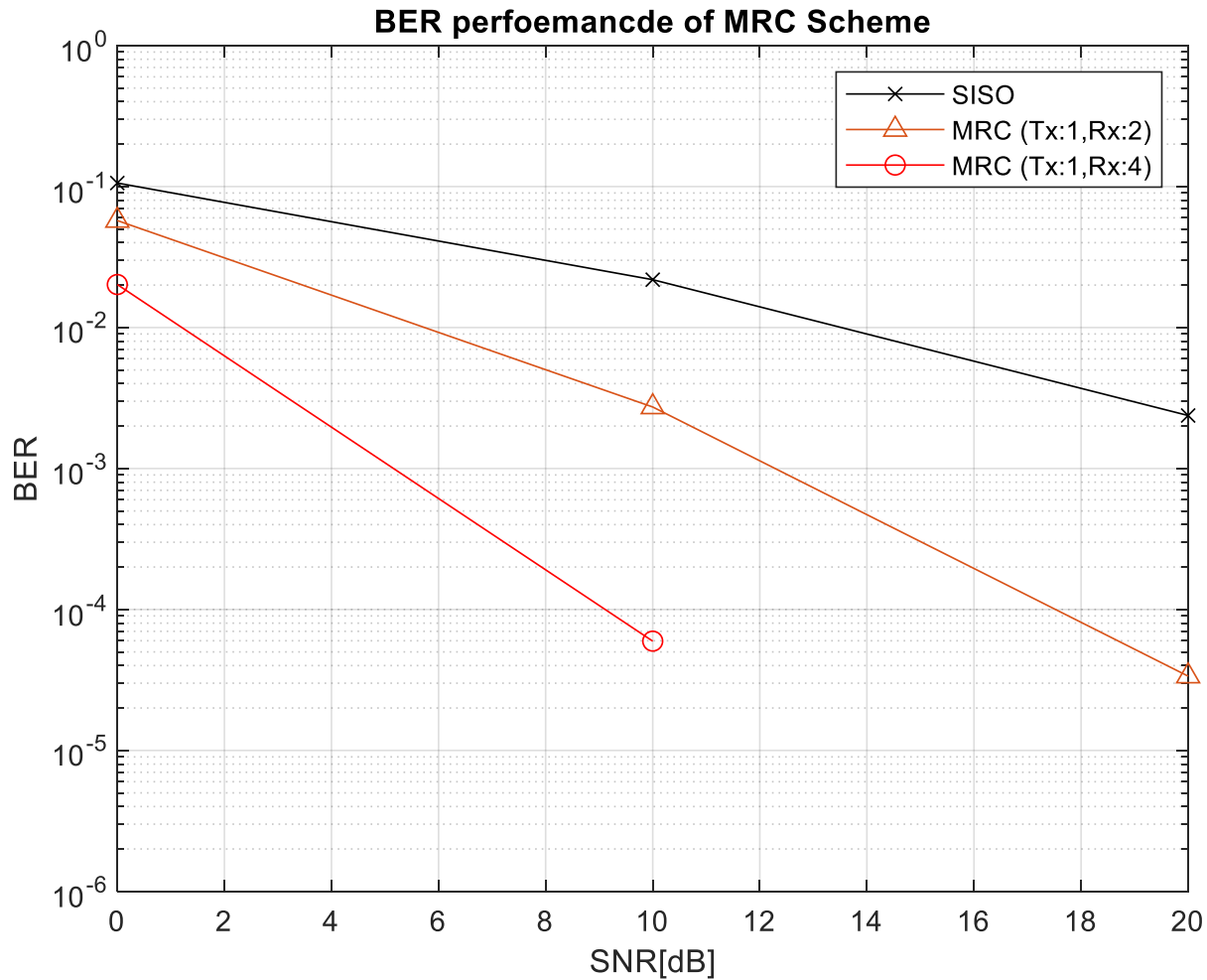
$$y_{MRC} = \underbrace{\left[w_1^{(MRC)} w_2^{(MRC)} \cdots w_{N_R}^{(MRC)} \right]}_{\mathbf{w}_{MRC}^T} \mathbf{y} = \sum_{i=1}^{N_R} w_i^{(MRC)} y_i \quad (10.5)$$

$$\begin{aligned} y_{MRC} &= \mathbf{w}_{MRC}^T \left(\sqrt{\frac{E_x}{N_0}} \mathbf{h}x + \mathbf{z} \right) \\ &= \sqrt{\frac{E_x}{N_0}} \mathbf{w}_{MRC}^T \mathbf{h}x + \mathbf{w}_{MRC}^T \mathbf{z} \end{aligned} \quad (10.6)$$

Kodlar

- Equal Gain Combining (EGC)
 - MRC'nin özel hali. Bütün katsayılar eşit
- MRC en iyi başarıyı elde eder.
 - MRC bir tür uzamsal uyumlu filtredir. $\mathbf{w}_{MRC} = \mathbf{h}^*$
- Kodlar
 - Program 10.1 “MRC_scheme.m” for performance of MRC for Rayleigh fading channels
 - Program 10.2 “modulator” for BPSK, QPSK, 8-PSK, 16-QAM mapping function

MRC Performansi



10.2 Uzay Zaman Kodlama

- Bir verici çeşitlemesi türü
- Semboller T zaman boyunca N_T antene eşleniyor.
- T zaman diliminde N sembol , R=
- Alıcıda semboller N_R antene ulaşıyor

$$y_j^{(t)} = \sqrt{\frac{E_x}{N_0 N_T}} \left[h_{j1}^{(t)} h_{j2}^{(t)} \cdots h_{jN_T}^{(t)} \right] \underbrace{\begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \\ \vdots \\ x_{N_T}^{(t)} \end{bmatrix}}_{\mathbf{x}^{(t)}} + z_j^{(t)} \quad (10.13)$$

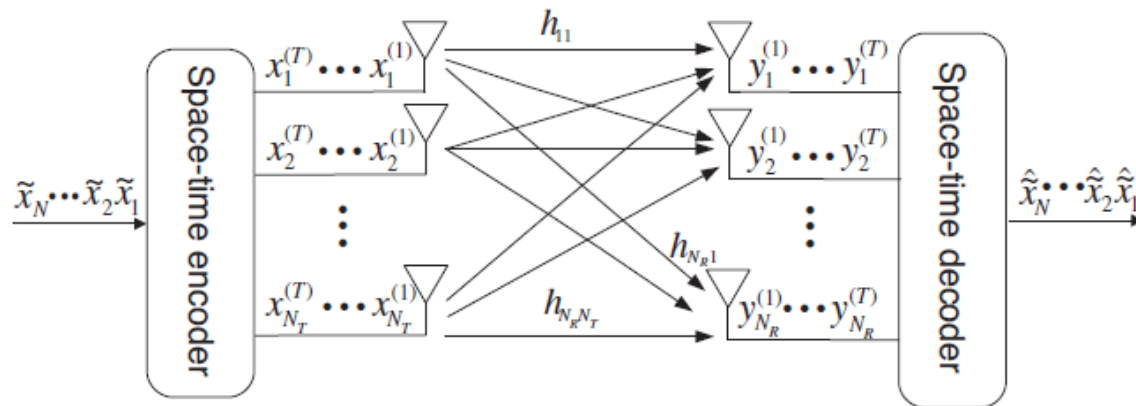


Figure 10.4 Space-time coded MIMO systems.

10.2.2 Pairwise Error Probability

$$\Pr(\mathbf{X} \rightarrow \mathbf{X}_{ML}) = Q\left(\sqrt{\frac{E_x}{2N_0N_T}} \|\mathbf{H}(\mathbf{X} - \mathbf{X}_{ML})\|_F\right) \quad (10.28)$$

$$\Pr(\mathbf{X} \rightarrow \mathbf{X}_{ML}) \leq \frac{1}{2} \exp\left(-\frac{E_x}{N_0N_T} \frac{\|\mathbf{H}(\mathbf{X} - \mathbf{X}_{ML})\|_F^2}{4}\right) \quad (10.29)$$

10.2.3 Uzay Zaman Kod Tasarımı

$$\begin{aligned}\|\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})\|_F^2 &= \text{tr}(\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H \mathbf{H}^H) \\ &= \sum_{l=1}^{N_R} \mathbf{h}_l(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H \mathbf{h}_l^H\end{aligned}\quad (10.30)$$

$$\sum_{l=1}^{N_R} \mathbf{h}_l(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^H \mathbf{h}_l^H = \sum_{l=1}^{N_R} \mathbf{h}_l \mathbf{V} \Lambda \mathbf{V}^H \mathbf{h}_l^H \quad (10.31)$$

- Diversity order: $N_R v < N_R N_T$

$$v = \min_{p \neq q} \text{rank} \{ (\mathbf{X}_p - \mathbf{X}_q)(\mathbf{X}_p - \mathbf{X}_q)^H \} \quad (10.39)$$

- Coding gain

$$\Lambda_{\min} = \min_{p \neq q} \prod_{i=1}^{N_T} \lambda_i(\mathbf{X}_p, \mathbf{X}_q). \quad (10.41)$$

Kodlama ve Çeşitleme Kazancı

- Çeşitleme kazancı: loglog BER grafiğinin eğimi
 - $T \geq N_T$ ise maksimum $N_R N_T$ olabilir. (rank kriteri)
- Kodlama kazancı: loglog BER grafiğinin sola kayması

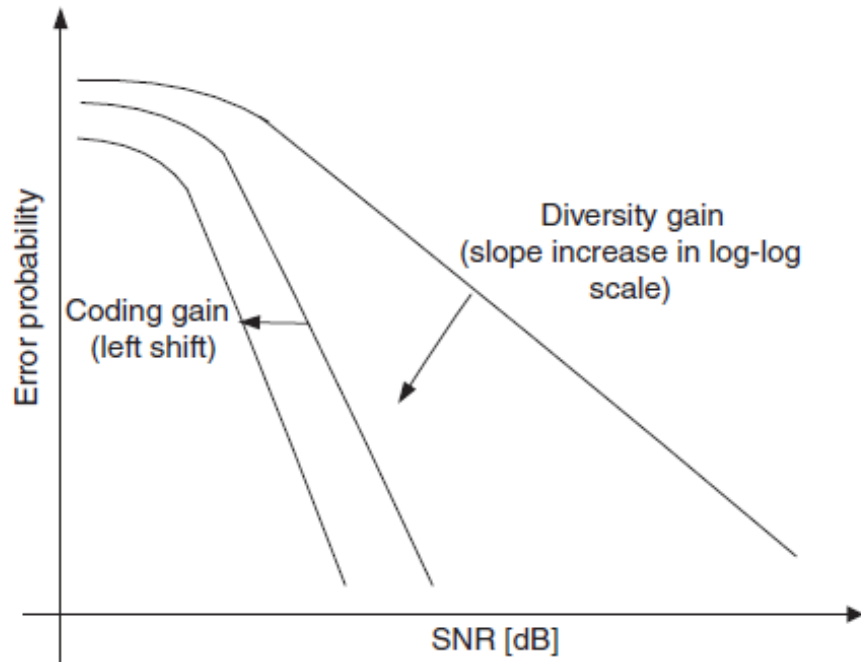


Figure 10.5 Diversity gain vs. coding gain.

10.3 Uzay Zaman Blok Kodu

10.3.1 Alamouti Kodu

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (10.42)$$

- $N_T = 2, T = 2, N = 2$
- Kod oranı 1

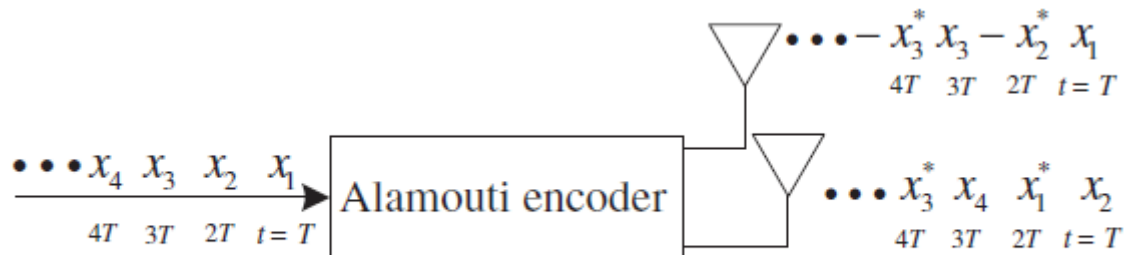


Figure 10.6 Alamouti encoder.

10.3.1 Alamouti Alıcısı

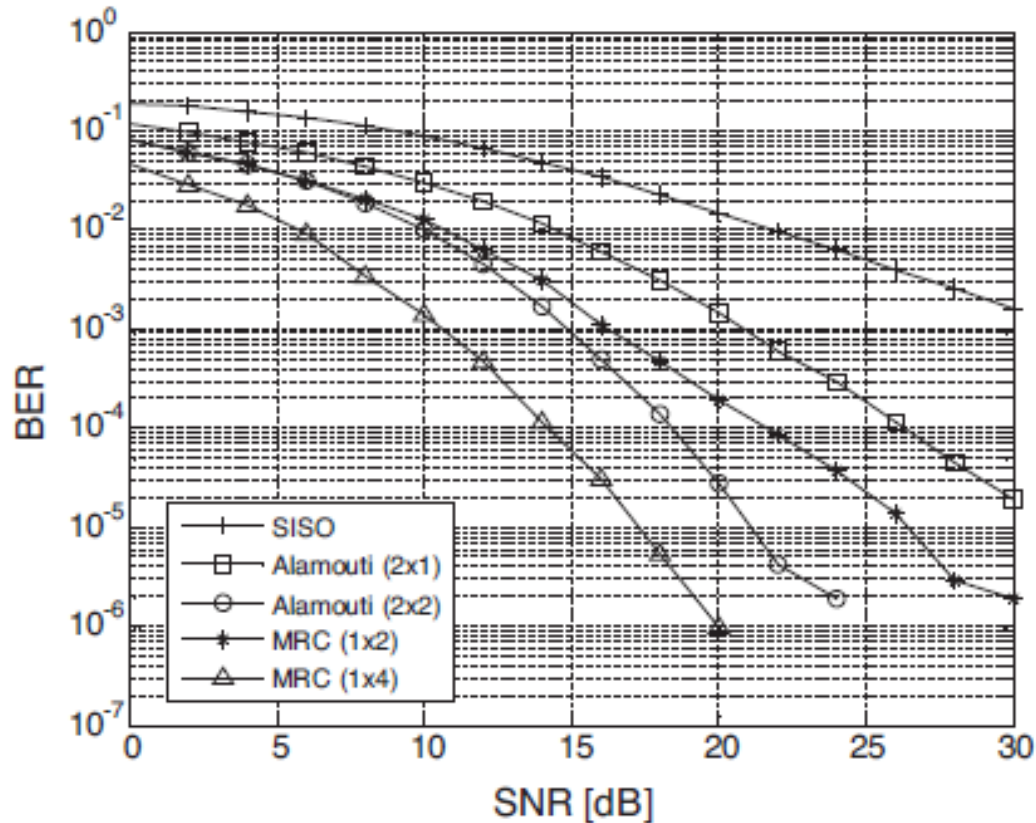
- Kanalin, iki sembol süresi boyunca sabit kaldığını varsayıyoruz.

$$\begin{aligned}y_1 &= h_1 x_1 + h_2 x_2 + z_1 \\y_2 &= -h_1 x_2^* + h_2 x_1^* + z_2\end{aligned}\tag{10.47}$$

$$\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} =$$

$$\hat{x}_{i,ML} = Q\left(\frac{\tilde{y}_i}{|h_1|^2 + |h_2|^2}\right), \quad i = 1, 2.\tag{10.52}$$

Performans

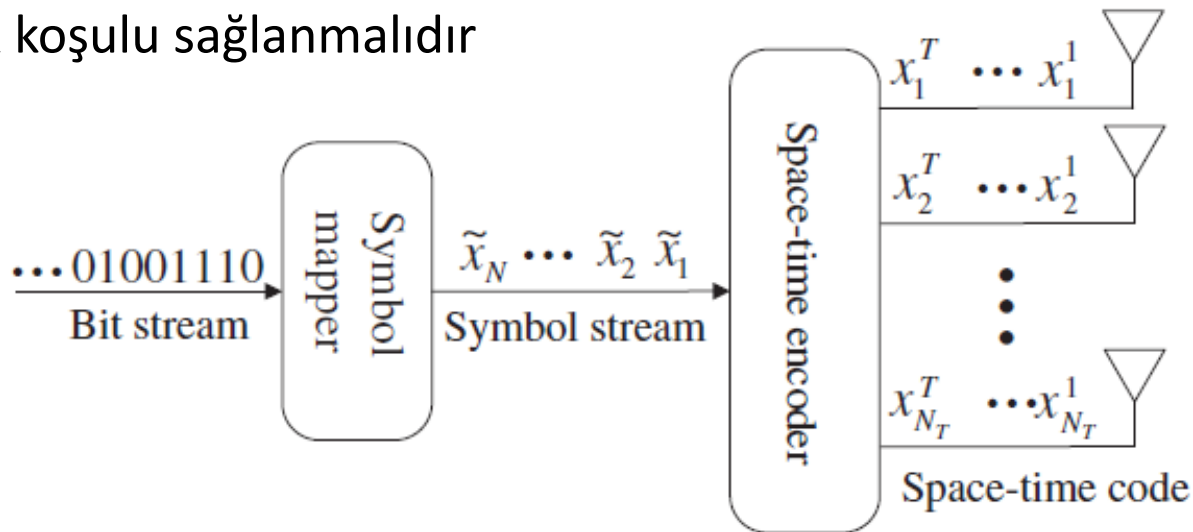


- Program 10.3 “Alamouti_scheme.m” for Alamouti space-time block coding
- MRC 1x2 ile Alamouti 2x1 aynı çeşitleme kazancı
- 2x2 Alamouti ile MRC 1x4 aynı çeşitleme kazancı

10.3.2 Genel Uzay Zaman Kodları

$$\begin{aligned} \mathbf{X}\mathbf{X}^H &= c \left(|x_i^1|^2 + |x_i^2|^2 + \cdots + |x_i^{N_T}|^2 \right) \mathbf{I}_{N_T} \\ &= c \|\mathbf{x}_i\|^2 \mathbf{I}_{N_T} \end{aligned} \quad (10.53)$$

- Diklik koşulu sağlanmalıdır



10.3.2.1 Gerçel Uzak Zaman Blok Kodları

$$\mathbf{X}_{2,\text{real}} = \begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} \quad (10.55)$$

$$\mathbf{X}_{4,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \quad (10.56)$$

$$\mathbf{X}_{8,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \quad (10.57)$$

10.3.2.1 Gerçel Uzak Zaman Blok Kodları

$$\mathbf{X}_{3,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix} \quad (10.59)$$

$$\mathbf{X}_{5,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \end{bmatrix} \quad (10.60)$$

$$\mathbf{X}_{6,\text{real}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \end{bmatrix} \quad (10.61)$$

10.3.2.1 Gerçel Uzak Zaman Blok Kodları

- Çeşitleme kazancı

10.3.2.2 Karmaşık Uzak Zaman Blok Kodları

- $N_T \geq 3$ için maksimum Çeşitleme kazancı ve kod oranı beraber sağlanamaz.
- $N_T \geq 2$: Alamouti kodu

$$\mathbf{X}_{2,\text{complex}} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad (10.64)$$

can know that there does not exist a complex space-time code

$$\mathbf{X}_{3,\text{complex}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \quad (10.65)$$

$$\mathbf{X}_{4,\text{complex}} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & x_3 & -x_2 & x_1 & x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \quad (10.66)$$

10.3.2.2 Karmaşık Uzak Zaman Blok Kodları

- Üstteki kodlar 1/2 oranlı
- Alttakiler daha yüksek oranlı ama daha fazla karmaşıklık var.

$$\mathbf{X}_{3,\text{complex}}^{\text{high rate}} = \begin{bmatrix} x_1 & -x_2 & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1 & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{bmatrix} \quad (10.67)$$

$$\mathbf{X}_{4,\text{complex}}^{\text{high rate}} = \begin{bmatrix} x_1 & -x_2 & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1 & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \\ \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} & \frac{(-x_2 - x_2^* + x_1 - x_1^*)}{2} & \frac{-(x_1 + x_1^* + x_2 - x_2^*)}{2} \end{bmatrix} \quad (10.68)$$

10.3.3 Uzay Zaman Kod Çözümü

$$[y_1 \ y_2 \ y_3 \ y_4] = \sqrt{\frac{E_x}{4N_0}} [h_1 \ h_2 \ h_3 \ h_4] \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} + [z_1 \ z_2 \ z_3 \ z_4], \quad (10.69)$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}_{\mathbf{y}_{\text{eff}}} = \sqrt{\frac{E_x}{4N_0}} \underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \end{bmatrix}}_{\mathbf{H}_{\text{eff}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}_{\text{eff}}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_3 \end{bmatrix}}_{\mathbf{z}_{\text{eff}}} \quad (10.70)$$

- Tek alıcı, 4 verici anten için ve gerçel bir STBC için

10.3.3 Uzay Zaman Kod Çözümü

$$\begin{aligned}
 [y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \ y_8] &= \sqrt{\frac{E_x}{3N_0}} [h_1 \ h_2 \ h_3] \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & x_1^* & -x_2^* & -x_3^* & -x_4^* \\ x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & -x_3^* \\ x_3 & -x_4 & x_1 & x_2 & x_3^* & -x_4^* & x_1^* & x_2^* \end{bmatrix} \\
 &+ [z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6 \ z_7 \ z_8]
 \end{aligned} \tag{10.73}$$

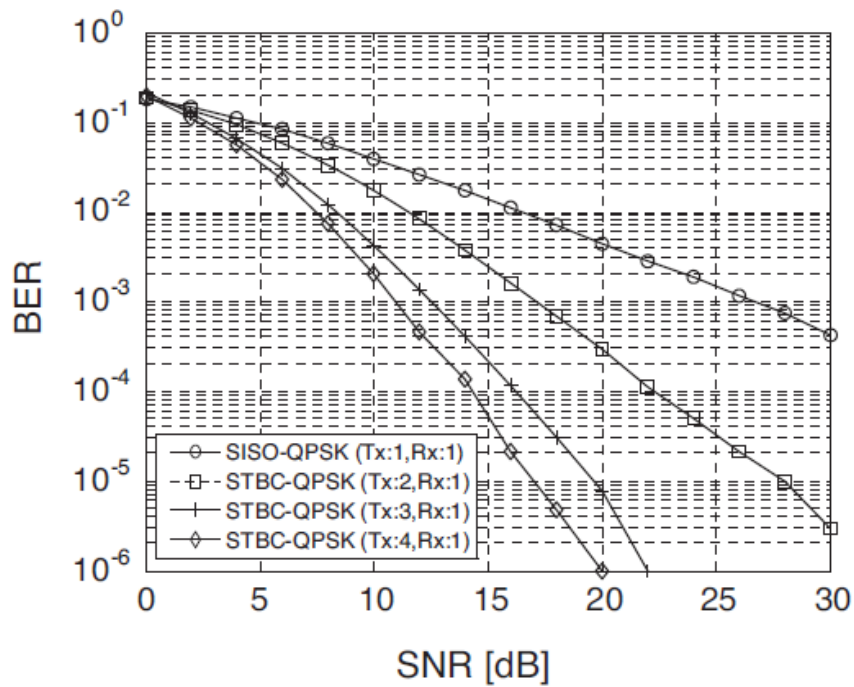
$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5^* \\ y_6^* \\ y_7^* \\ y_8^* \end{bmatrix}}_{y_{\text{eff}}} = \sqrt{\frac{E_x}{3N_0}} \underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & -h_1 \\ h_1^* & h_2^* & h_3^* & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3^* & 0 & -h_1^* & h_2^* \\ 0 & h_3^* & -h_2^* & -h_1^* \end{bmatrix}}_{\mathbf{H}_{\text{eff}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x_{\text{eff}}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5^* \\ z_6^* \\ z_7^* \\ z_8^* \end{bmatrix}}_{z_{\text{eff}}} \tag{10.74}$$

- Tek alıcı, üç verici anten için ve karmaşık bir STBC için

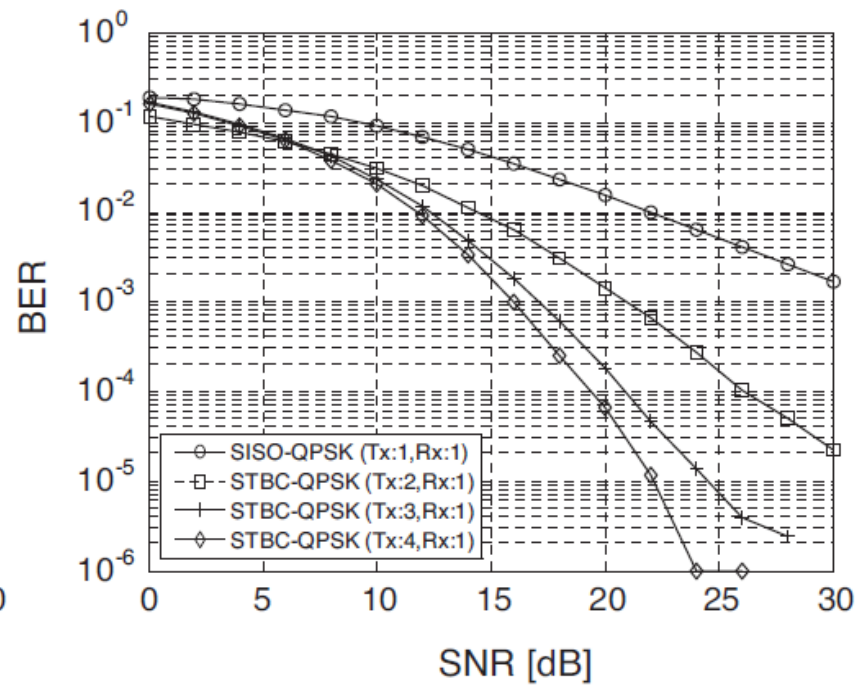
10.3.3 Uzay Zaman Kod Çözümü

$$\begin{aligned}
 [y_1 \ y_2 \ y_3 \ y_4] &= \sqrt{\frac{E_x}{3N_0}} [h_1 \ h_2 \ h_3] \begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{bmatrix} \\
 &+ [z_1 \ z_2 \ z_3 \ z_4]
 \end{aligned}
 \tag{10.77}$$

- Tek alıcı, üç verici anten için ve karmaşık, yüksek oranlı bir STBC için
- (10.77), (10.78), (10.79), (10.80), (10.81),
- Daha fazla denklem manipülasyonu yapmak gerekiyor.
- Sonuçta yine denklemler oluşuyor
- Program 10.4 “STBC_3x4_simulation.m” to simulate 3x4 Space-Time Block Coding



(a) QPSK



(b) 16-QAM

Figure 10.9 BER performance of various space-time block codes.