MIMO-OFDM Wireless Communications with MATLAB®

Chapter 11. Uzamsal Çoklamalı MIMO Sistemleri

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WILEY

Chapter 11. Signal Detection for Spatially Multiplexed MIMO Systems

Bu konudaki varsayım CSIR

Sadece alıcıda kanal bilgisi olduğu varsayımı

(Linear)

(expo-complex)

Verici her antenden farklı bilgi basar.

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Chapter 11. Signal Detection for Spatially Multiplexed MIMO System

- 11.7 SOFT DECISION FOR MIMO SYSTEMS
  - 11.7.1 Log-likelihood-Ratio (LLR) for SISO Systems
  - 11.7.2 LLR for Linear Detector based MIMO System
  - 11.7.3 LLR for MIMO System with a Candidate Vector Set
  - 11.7.4 LLR for MIMO System using a Limited Candidate Vector Set

Chapter 11. Uzamsal Çoklamalı MIMO Sistemleri

$N_R \times N_T$ $N_T \times 1$ vector $\rightarrow [x_1, x_2, \ldots, x_{N_T}]^T$

$N_R \times 1$ noise $[z_1, z_2, \ldots z_{N_R}]^T \sim \mathcal{N}(0, \sigma_z^2)$

rx $[y_1 y_2, \ldots y_{N_R}]^T$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \cdots + \mathbf{h}_{N_T} x_{N_T} + \mathbf{z} \quad (11.1)$$

$N_R \times 1$ $N_R \times 1$ $N_R \times 1$ skaler

11.1 Linear Signal Detection

istenen sinyal dışında her şey gürültü muamelesi görür

$$\tilde{\mathbf{x}} = [\tilde{x}_1 \tilde{x}_2 \cdots \tilde{x}_{N_T}]^T = \mathbf{W}\mathbf{y}, \quad (11.2)$$

$N_R \times 1$

$N_T \times N_R$ weight matrix

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11.1 Linear Signal Detection

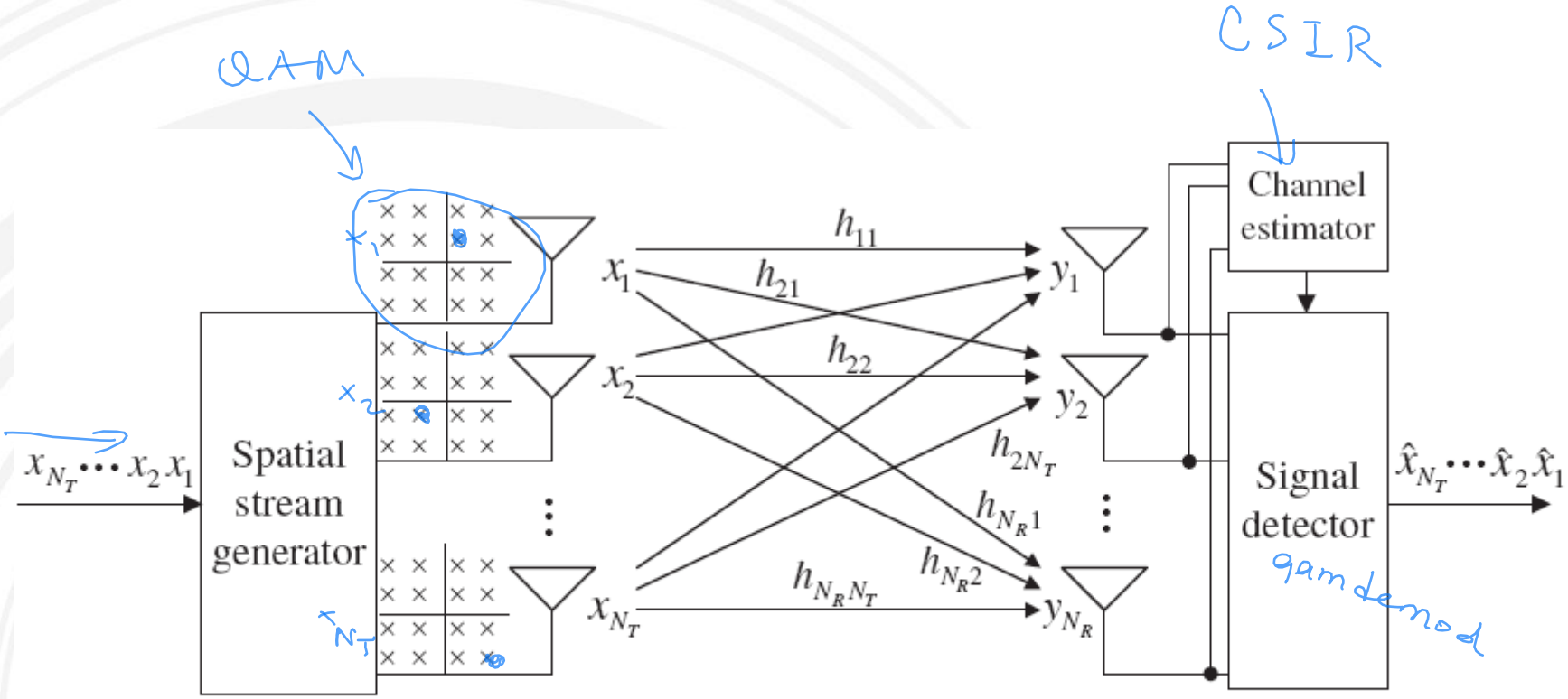

Figure 11.1 Spatially multiplexed MIMO systems.

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11.1.1 ZF Signal Detection (Zero Forcing)

$$\mathbf{W}_{ZF} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H. \tag{11.3}$$

$$\begin{aligned}\tilde{\mathbf{x}}_{ZF} &= \mathbf{W}_{ZF}\mathbf{y} \\ &= \mathbf{x} + \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{z} \\ &= \mathbf{x} + \tilde{\mathbf{z}}_{ZF}\end{aligned} \tag{11.4}$$

$$\begin{aligned}\|\tilde{\mathbf{z}}_{ZF}\|_2^2 &= \left\|\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{z}\right\|^2 \\ &= \left\|\left(\mathbf{V}\boldsymbol{\Sigma}^2\mathbf{V}^H\right)^{-1}\mathbf{V}\boldsymbol{\Sigma}\mathbf{U}^H\mathbf{z}\right\|^2 \\ &= \left\|\mathbf{V}\boldsymbol{\Sigma}^{-2}\mathbf{V}^H\mathbf{V}\boldsymbol{\Sigma}\mathbf{U}^H\mathbf{z}\right\|^2 \\ &= \left\|\mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\right\|^2\end{aligned} \tag{11.5}$$

11.1.1 ZF Signal Detection

$$
\begin{aligned}
E\left\{\|\tilde{\mathbf{z}}_{ZF}\|_2^2\right\} &= E\left\{\left\|\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\right\|_2^2\right\} \\
&= E\left\{\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{z}\mathbf{z}^H\mathbf{U}\boldsymbol{\Sigma}^{-1}\right)\right\} \\
&= \mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathrm{E}\left\{\mathbf{z}\mathbf{z}^H\right\}\mathbf{U}\boldsymbol{\Sigma}^{-1}\right) \\
&= \mathrm{tr}\left(\sigma_z^2\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}^{-1}\right) \\
&= \sigma_z^2\mathrm{tr}\left(\boldsymbol{\Sigma}^{-2}\right) \\
&= \sum_{i=1}^{N_T}\frac{\sigma_z^2}{\sigma_i^2}.
\end{aligned}
\tag{11.6}
$$

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11.1.2 MMSE Signal Detection

$$\mathbf{W}_{MMSE} = \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H. \tag{11.7}$$

$$\mathbf{w}_{i,MMSE} = \underset{\mathbf{w}=(w_1,w_2,\ldots,w_{N_T})}{\arg\max} \frac{\left|\mathbf{w}\mathbf{h}_i\right|^2 \mathsf{E}_x}{\mathsf{E}_x \sum_{j=1, j\neq i}^{N_T} \left|\mathbf{w}\mathbf{h}_j\right|^2 + \|\mathbf{w}\|^2\sigma_z^2} \tag{11.8}$$

$$\begin{aligned}\tilde{\mathbf{x}}_{MMSE} &= \mathbf{W}_{MMSE}\mathbf{y} \\ &= \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{y} \\ &= \tilde{\mathbf{x}} + \left(\mathbf{H}^H\mathbf{H} + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{z} \\ &= \tilde{\mathbf{x}} + \tilde{\mathbf{z}}_{MMSE}\end{aligned} \tag{11.9}$$

11.1.2 MMSE Signal Detection

$$\begin{aligned}\|\tilde{\mathbf{z}}_{MMSE}\|_2^2 &= \left\|\left(\mathbf{H}^H\mathbf{H}+\sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{z}\right\|^2 \\ &= \left\|\left(\mathbf{V}\boldsymbol{\Sigma}^2\mathbf{V}^H+\sigma_z^2\mathbf{I}\right)^{-1}\mathbf{V}\boldsymbol{\Sigma}\mathbf{U}^H\mathbf{z}\right\|^2.\end{aligned} \tag{11.10}$$

$$\|\tilde{\mathbf{z}}_{MMSE}\|_2^2 = \left\|\left(\boldsymbol{\Sigma}\mathbf{V}^H+\sigma_z^2\boldsymbol{\Sigma}^{-1}\mathbf{V}^H\right)^{-1}\mathbf{U}^H\mathbf{z}\right\|^2 = \left\|\mathbf{V}\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z}\right\|^2 \tag{11.11}$$

11.1.2 MMSE Signal Detection

$$
\begin{aligned}
E\left\{\|\tilde{\mathbf{z}}_{MMSE}\|_2^2\right\} &= E\left\{\left\|\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z}\right\|^2\right\} \\
&= E\left\{\mathrm{tr}\left(\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H\mathbf{z}\mathbf{z}^H\mathbf{U}\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\right)\right\} \\
&= \mathrm{tr}\left(\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\mathbf{U}^H E\{\mathbf{z}\mathbf{z}^H\}\mathbf{U}\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\right) \\
&= \mathrm{tr}\left(\sigma_z^2\left(\boldsymbol{\Sigma}+\sigma_z^2\boldsymbol{\Sigma}^{-1}\right)^{-2}\right) \qquad (11.12) \\
&= \sum_{i=1}^{N_T}\sigma_z^2\left(\sigma_i+\frac{\sigma_z^2}{\sigma_i}\right)^{-2} \\
&= \sum_{i=1}^{N_T}\frac{\sigma_z^2\sigma_i^2}{\left(\sigma_i^2+\sigma_z^2\right)^2}.
\end{aligned}
$$

11.1.2 MMSE Signal Detection

$$E\left\{\|\tilde{\mathbf{z}}_{ZF}\|_2^2\right\} = \sum_{i=1}^{N_T} \frac{\sigma_z^2}{\sigma_i^2} \approx \frac{\sigma_z^2}{\sigma_{\min}^2} \quad \text{for ZF} \qquad (11.13a)$$

$$E\left\{\|\tilde{\mathbf{z}}_{MMSE}\|_2^2\right\} = \sum_{i=1}^{N_T} \frac{\sigma_z^2\sigma_i^2}{\left(\sigma_i^2+\sigma_z^2\right)^2} \approx \frac{\sigma_z^2\sigma_{\min}^2}{\left(\sigma_{\min}^2+\sigma_z^2\right)^2} \quad \text{for MMSE} \qquad (11.13b)$$

11.2 OSIC Signal Detection

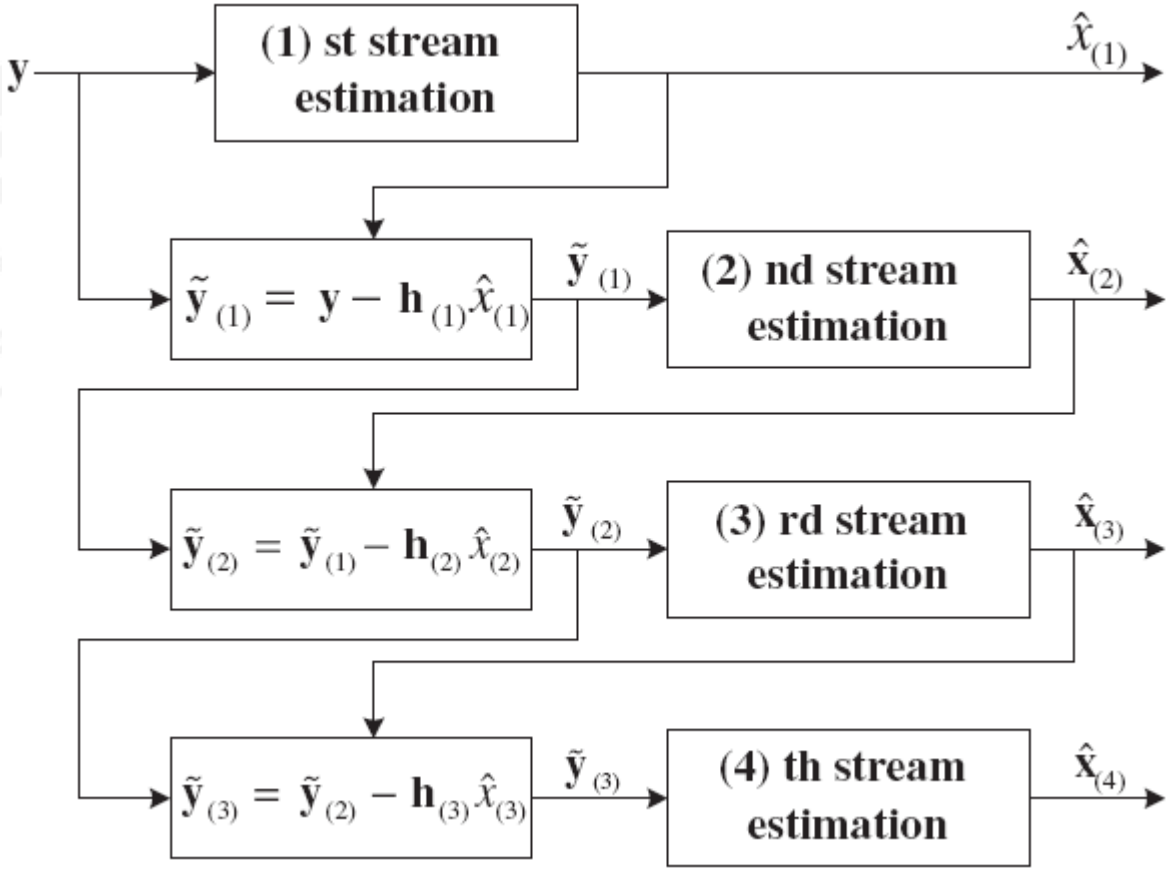

Figure 11.2 Illustration of OSIC signal detection for four spatial streams (i.e., $N_T = 4$).

11.2 OSIC Signal Detection

$$\begin{aligned}\tilde{\mathbf{y}}_{(1)} &= \mathbf{y}-\mathbf{h}_{(1)}\hat{x}_{(1)} \\ &= \mathbf{h}_{(1)}\left(x_{(1)}-\hat{x}_{(1)}\right)+\mathbf{h}_{(2)}x_{(2)}+\cdots+\mathbf{h}_{(N_T)}x_{(N_T)}+\mathbf{z}.\end{aligned} \tag{11.14}$$

$$\mathrm{SINR}_i = \frac{\mathsf{E}_x\left|\mathbf{w}_{i,MMSE}\mathbf{h}_i\right|^2}{\mathsf{E}_x\sum_{l\neq i}\left|\mathbf{w}_{i,MMSE}\mathbf{h}_l\right|+\sigma_z^2\left\|\mathbf{w}_{i,MMSE}\right\|^2}, \quad i=1,2,\ldots,N_T \tag{11.15}$$

$$\mathbf{H}^{(1)} = [\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_{l-1}\ \mathbf{h}_{l+1}\ \cdots\ \mathbf{h}_{N_T}] \tag{11.16}$$

$$\mathrm{SNR}_i = \frac{\mathsf{E}_x}{\sigma_z^2\left\|\mathbf{w}_i\right\|^2}, \quad i=1,2,\ldots,N_T. \tag{11.17}$$

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{z}=\mathbf{h}_1x_1+\mathbf{h}_2x_2+\cdots+\mathbf{h}_{N_T}x_{N_T}+\mathbf{z} \tag{11.18}$$

11.2 OSIC Signal Detection

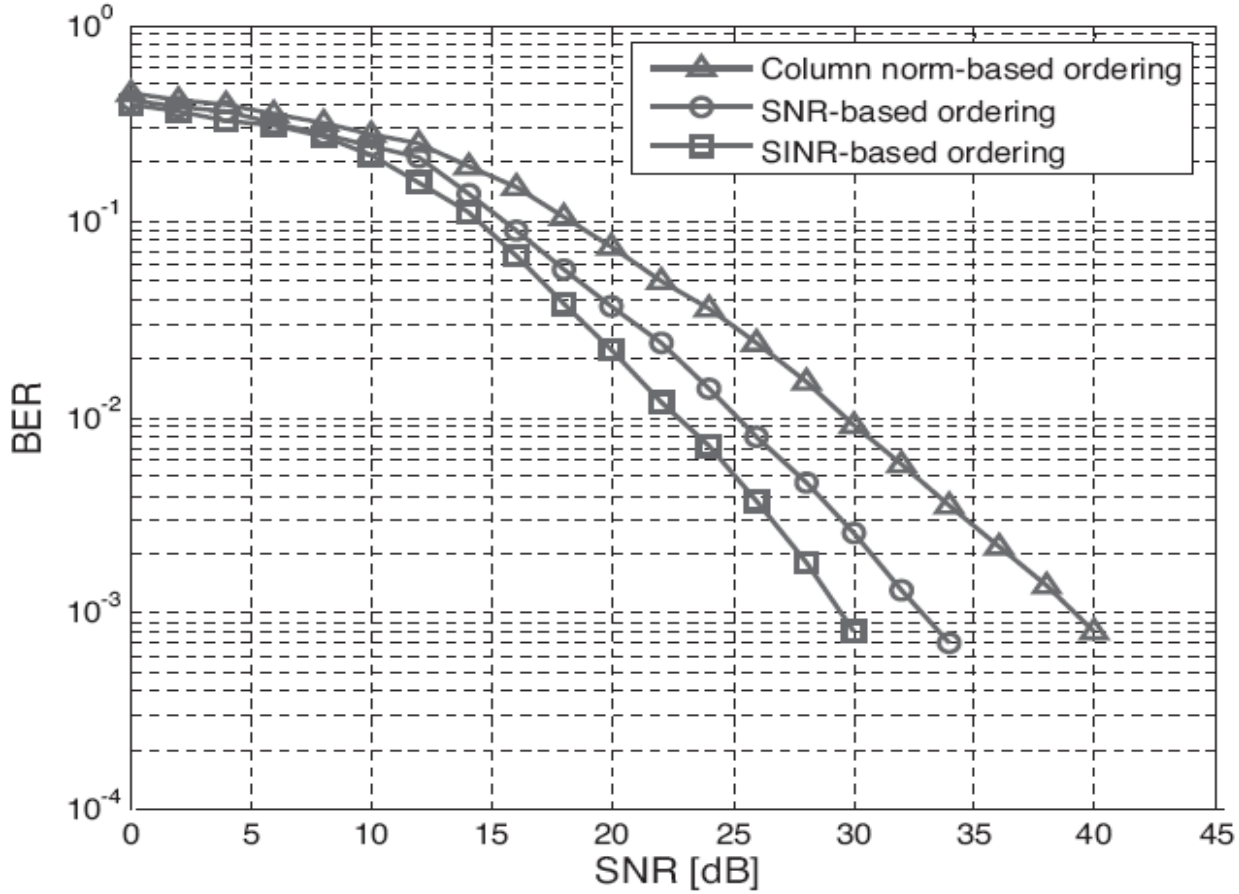

Figure 11.3 Performance of OSIC methods with different detection ordering.

Programlar

- Program 11.1 “OSIC_detector” implementing the various OSIC signal detection methods
- Program 11.2 “QAM16_slicer”
- Program 11.3 “ML_detector” for ML signal detection

11.3 ML Signal Detection

- Alınan sinyal vektörü (**y**)
- Gönderilmesi olası bütün sinyal vektörleri $\boldsymbol{x} \in C^{N_T}$ için kanal matrisi ile çarpım. $\boldsymbol{Hx}$
- Bunlar arasındaki uzaklığı minimize eden $\boldsymbol{x}$ değeri.

$$\hat{\mathbf{x}}_{ML} = \underset{\mathbf{x} \in \mathbb{C}^{N_T}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \qquad (11.19)$$

- Bütün vektörler eşit olasılıklı ise bu yöntem optimaldir.
- Bit/sembol (k) ve verici anten sayısı arttıkça karmaşıklık üstel artar $O(2^{kN_T})$

11.3 ML Signal Detection

$$\hat{\mathbf{x}}_{ML} = \underset{\mathbf{x}\in \mathbb{C}^{N_T}}{\operatorname{argmin}} \|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2 \qquad (11.19)$$

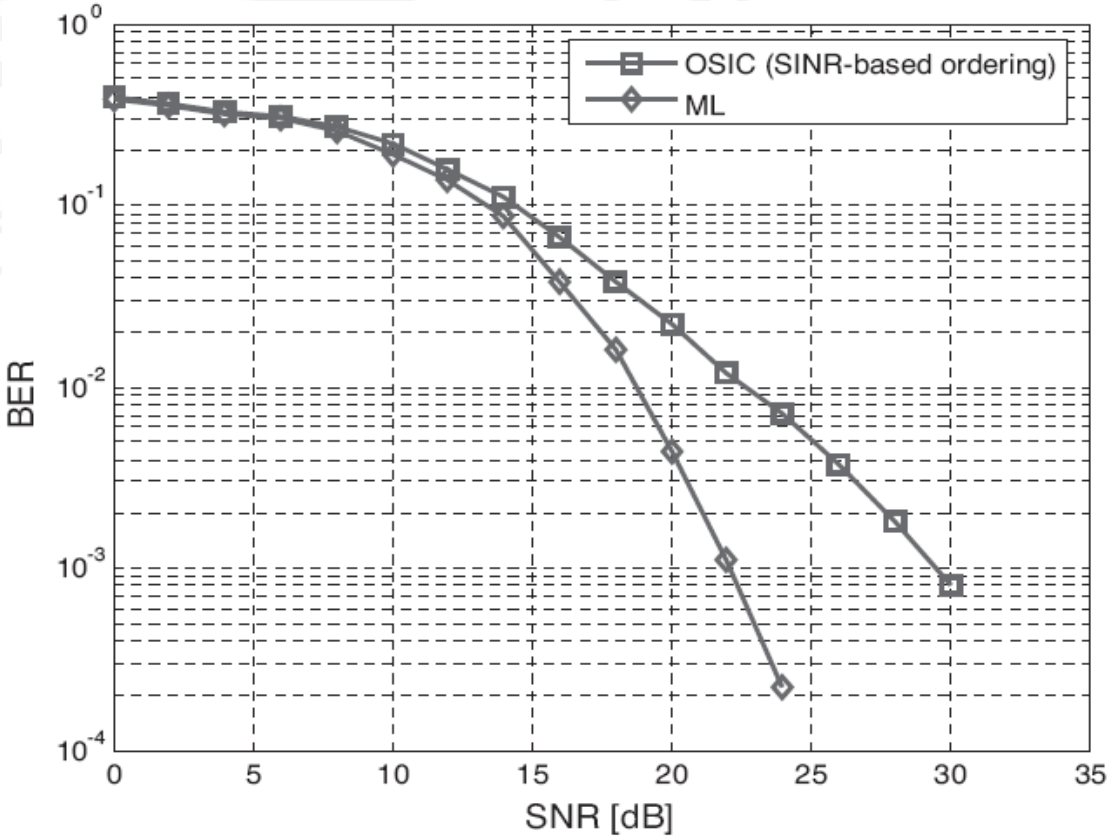

Figure 11.4 Performance comparison: OSIC vs. ML detection methods.