MIMO-OFDM Wireless Communications with MATLAB®

Chapter 13. Çok Kullanıcılı MIMO (MU-MIMO)

Yong Soo Cho | Jaekwon Kim
Won Young Yang | Chung G. Kang

WILEY

Chapter 13. Çok Kullanıcılı MIMO

- 13.1 ÇOK KULLANICILI MIMO MATEMATİKSEL MODELİ
- 13.2 ÇOK KULLANICILI MIMO KANAL KAPASITESİ
  - 13.2.1 MAC Kapasitesi
  - 13.2.2 BC Kapasitesi
- 13.3 YAYIN KANALI İLETİM METOTLARI
  - 13.3.1 **Channel Inversion Methods**
  - 13.3.2 **Block Diagonalization Method**
  - 13.3.3 Dirty Paper Coding (DPC)
  - 13.3.4 Tomlinson-Harashima Precoding

Chapter 13. Çok Kullanıcılı MIMO Giriş

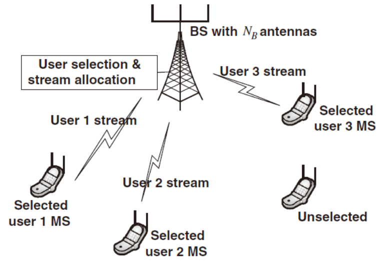

Figure 13.1 Multi-user MIMO communication systems: $K = 4$.

Chapter 13. Çok Kullanıcılı MIMO

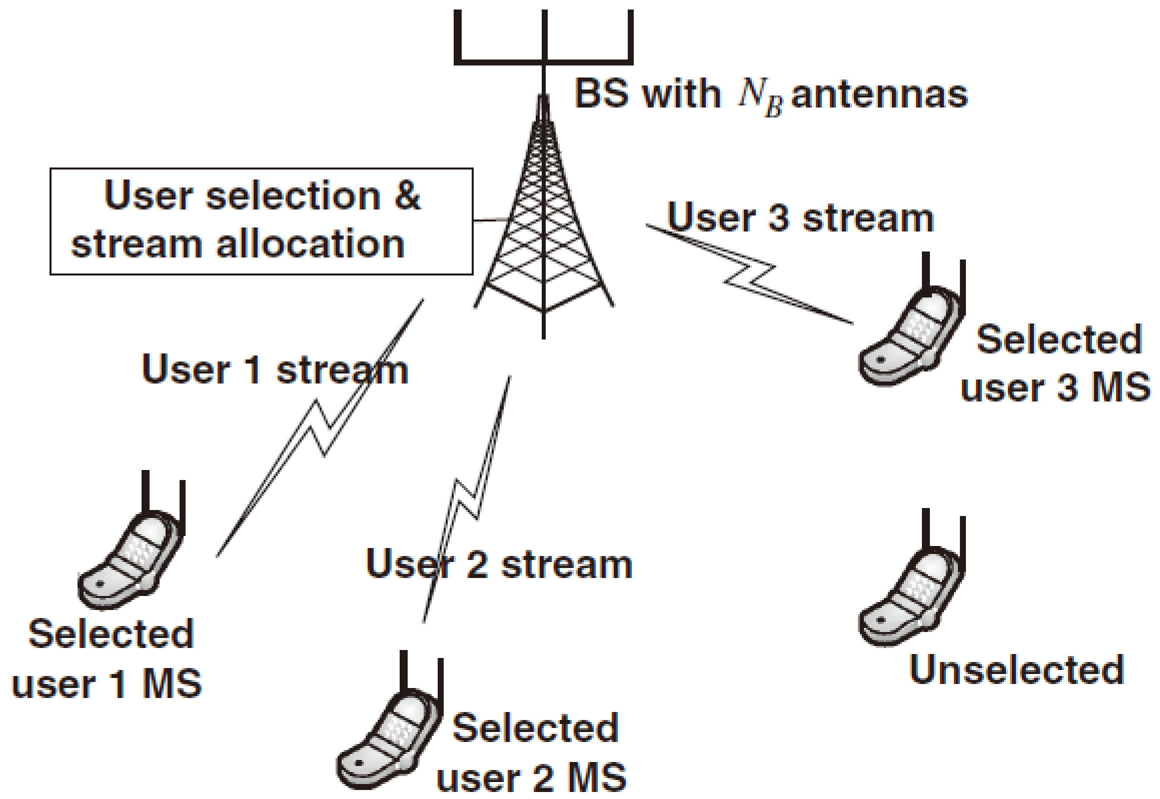

Figure 13.1 Multi-user MIMO communication systems: $K = 4$.

Chapter 13. Çok Kullanıcılı MIMO
13.1 Çok Kullanıcılı MIMO Matematiksel Modeli

$$\begin{aligned}\mathbf{y}_{MAC} &= \mathbf{H}_1^{\mathrm{UL}}\mathbf{x}_1 + \mathbf{H}_2^{\mathrm{UL}}\mathbf{x}_2 + \cdots + \mathbf{H}_K^{\mathrm{UL}}\mathbf{x}_K + \mathbf{z} \\ &= \underbrace{\left[\mathbf{H}_1^{\mathrm{UL}}\,\mathbf{H}_2^{\mathrm{UL}}\cdots\mathbf{H}_K^{\mathrm{UL}}\right]}_{=\mathbf{H}^{\mathrm{UL}}}\begin{bmatrix}\mathbf{x}_1\\ \vdots \\ \mathbf{x}_K\end{bmatrix} + \mathbf{z} = \mathbf{H}^{\mathrm{UL}}\begin{bmatrix}\mathbf{x}_1\\ \vdots \\ \mathbf{x}_K\end{bmatrix} + \mathbf{z}\end{aligned} \tag{13.1}$$

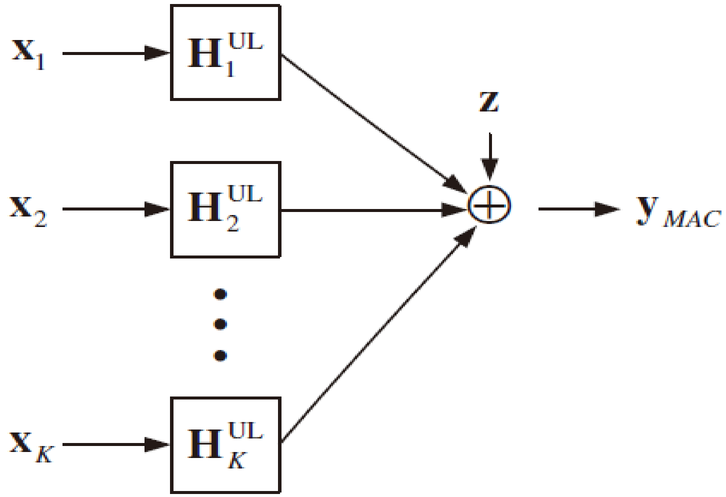

Figure 13.2 Uplink channel model for multi-user MIMO system: multiple access channel (MAC).

13.1 Çok Kullanıcılı MIMO Matematiksel Modeli

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{DL}}\mathbf{x} + \mathbf{z}_u, \quad u = 1, 2, \cdots, K \tag{13.2}$$

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix}}_{\mathbf{y}_{BC}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \vdots \\ \mathbf{H}_K^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}_{\mathrm{DL}}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_K \end{bmatrix}}_{\mathbf{z}} \tag{13.3}$$

13.1 Çok Kullanıcılı MIMO Matematiksel Modeli

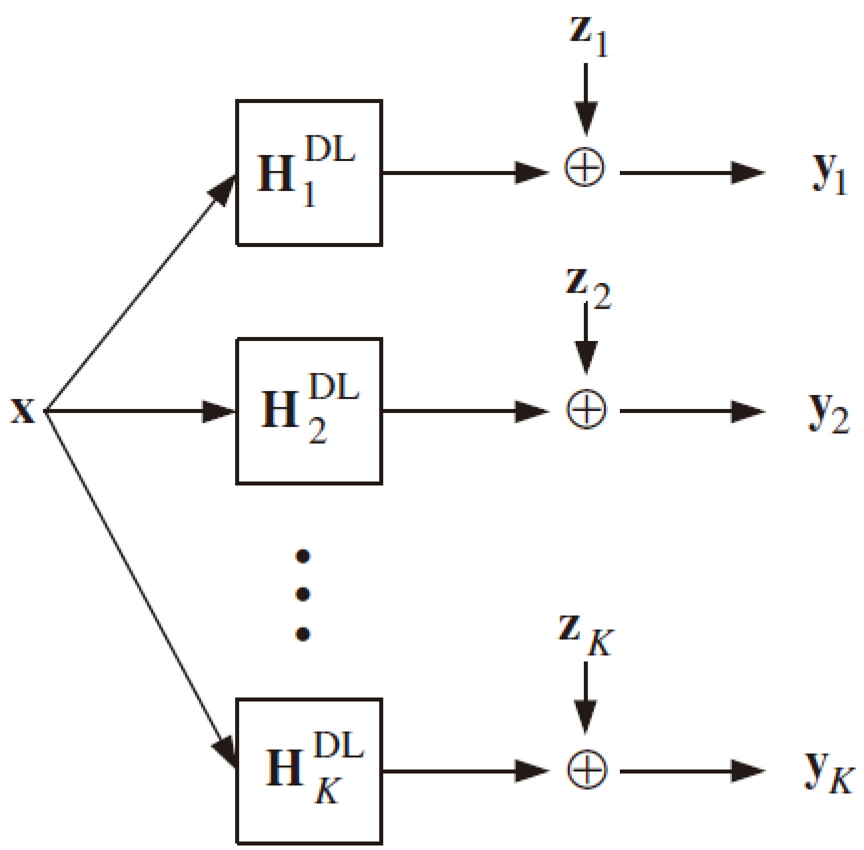

Figure 13.3 Downlink channel model for multi-user MIMO system: broadcast channel (BC).

13.2 Çok kullanıcılı MIMO Kanal kapasitesi
13.2.1 MAC Kapasitesi

$$R_1 \leq \log_2\left(1 + \left\|\mathbf{H}_1^{\mathrm{UL}}\right\|^2 P_1\right)$$

$$R_2 \leq \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{UL}}\right\|^2 P_2\right) \qquad (13.4)$$

$$R_1 + R_2 \leq \log_2\left(1 + \left\|\mathbf{H}_1^{\mathrm{UL}}\right\|^2 P_1 + \left\|\mathbf{H}_2^{\mathrm{UL}}\right\|^2 P_2\right)$$

$$\begin{aligned} \mathbf{y}_{MAC} &= \mathbf{H}_1^{\mathrm{UL}} x_1 + \mathbf{H}_2^{\mathrm{UL}} x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ &= \left[\mathbf{H}_1^{\mathrm{UL}}\ \mathbf{H}_2^{\mathrm{UL}}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \end{aligned} \qquad (13.5)$$

$$\tilde{\mathbf{y}}_{MAC} = \mathbf{y}_{MAC} - \mathbf{H}_1^{\mathrm{UL}} x_1 = \mathbf{H}_2^{\mathrm{UL}} x_2 + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (13.6)$$

13.2.1 MAC Kapasitesi

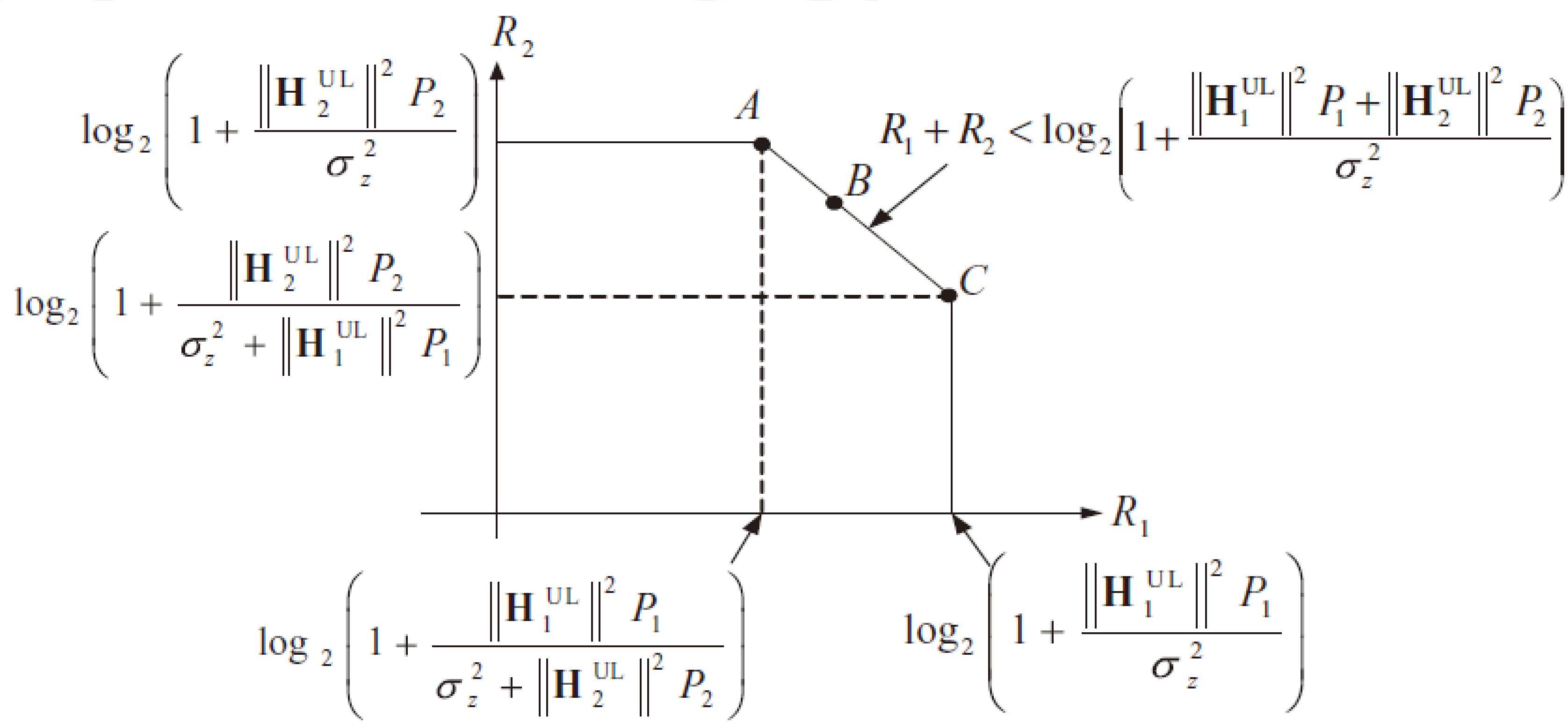

Figure 13.4 Capacity region of MAC: $K = 2$ and $N_M = 1$.

MIMO-OFDM Wireless Communications with MATLAB®
Yong Soo Cho, Jaekwon Kim, Won Young Yang, and Chung G. Kang

13.2.2 Yayın Kanalı Kapasitesi

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{y}_{BC}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{DL} \\ \mathbf{H}_2^{DL} \end{bmatrix}}_{\mathbf{H}^{DL}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (13.7)$$

$$\mathbf{H}^{DL} = \underbrace{\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix}}_{\mathbf{Q}} \qquad (13.8)$$

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} = \mathbf{Q}^H \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \frac{1}{l_{22}} l_{21} \tilde{x}_1 \end{bmatrix} \qquad (13.9)$$

13.2.2 Yayın Kanalı Kapasitesi

$$
\begin{aligned}
\mathbf{y}_{\mathrm{BC}} &= \mathbf{H}^{\mathrm{DL}}\mathbf{x}+\mathbf{z} \\
&= \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} \left[\mathbf{q}_1^H \; \mathbf{q}_2^H\right] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \dfrac{1}{l_{22}} l_{21}\tilde{x}_1 \end{bmatrix} + \mathbf{z} \\
&= \begin{bmatrix} l_{11} & 0 \\ 0 & l_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \mathbf{z} \\
&= \begin{bmatrix} \left\|\mathbf{H}_1^{\mathrm{DL}}\right\| & 0 \\ 0 & \left\|\mathbf{H}_2^{\mathrm{DL}} - l_{12}\mathbf{q}_1\right\| \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \mathbf{z}
\end{aligned}
\tag{13.10}
$$

13.2.2 Yayın Kanalı Kapasitesi

$$R_1 = \log\left(1 + \left\|\mathbf{H}_1^{\mathrm{DL}}\right\|^2 \frac{\alpha P}{\sigma_z^2}\right), \qquad (13.11)$$

$$R_2 = \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{DL}} - l_{21}\mathbf{q}_1\right\|^2 \frac{(1-\alpha)P}{\sigma_z^2}\right). \qquad (13.12)$$

$$R_2 = \log_2\left(1 + \left\|\mathbf{H}_2^{\mathrm{DL}}\right\|^2 \frac{(1-\alpha)P}{\sigma_z^2}\right). \qquad (13.13)$$

13.3 Yayın Kanalı İletim Yöntemi
13.3.1 Kanalın Tersini Alma

$$y_u = \mathbf{H}_u^{\mathrm{DL}} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_K \end{bmatrix} + z_u, \quad u = 1, 2, \cdots, K. \tag{13.14}$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix}}_{\mathbf{y}_{\mathrm{BC}}} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} \\ \vdots \\ \mathbf{H}_K^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}^{\mathrm{DL}}} \underbrace{\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_K \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}}_{\mathbf{z}} \tag{13.15}$$

13.3.1 Kanalın Tersini Alma

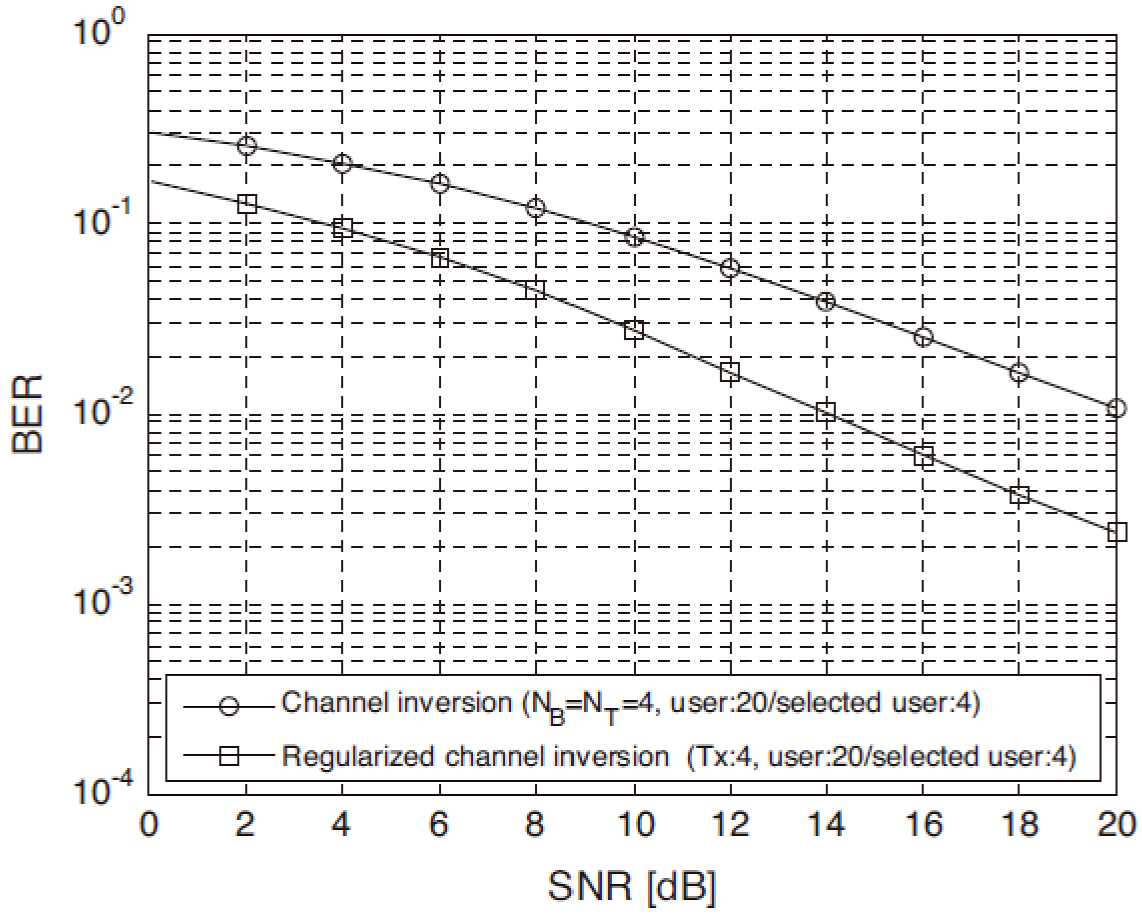

Figure 13.5 BER performance of two channel inversion methods.

Kodlar

- Program 13.1 “multi_user_MIMO.m” for a multi-userMIMOsystem with channel inversion
- Program 13.2 “QPSK_mapper”
- Program 13.3 “QPSK_slicer”

13.3.2 Block Diagonalization

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{DL}} \sum_{k=1}^{K} \mathbf{W}_k \tilde{\mathbf{x}}_k + \mathbf{z}_u = \mathbf{H}_u^{\mathrm{DL}} \mathbf{W}_u \tilde{\mathbf{x}}_u + \sum_{k=1,\, k \neq u}^{K} \mathbf{H}_u^{\mathrm{DL}} \mathbf{W}_k \tilde{\mathbf{x}}_k + \mathbf{z}_u \tag{13.16}$$

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} & \mathbf{H}_1^{\mathrm{DL}} & \mathbf{H}_1^{\mathrm{DL}} \\ \mathbf{H}_2^{\mathrm{DL}} & \mathbf{H}_2^{\mathrm{DL}} & \mathbf{H}_2^{\mathrm{DL}} \\ \mathbf{H}_3^{\mathrm{DL}} & \mathbf{H}_3^{\mathrm{DL}} & \mathbf{H}_3^{\mathrm{DL}} \end{bmatrix}}_{\mathbf{H}_{\mathrm{DL}}} \underbrace{\begin{bmatrix} \mathbf{W}_1 \tilde{\mathbf{x}}_1 \\ \mathbf{W}_2 \tilde{\mathbf{x}}_2 \\ \mathbf{W}_3 \tilde{\mathbf{x}}_3 \end{bmatrix}}_{\mathbf{x}} + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}} \mathbf{W}_1 & \mathbf{H}_1^{\mathrm{DL}} \mathbf{W}_2 & \mathbf{H}_1^{\mathrm{DL}} \mathbf{W}_3 \\ \mathbf{H}_2^{\mathrm{DL}} \mathbf{W}_1 & \mathbf{H}_2^{\mathrm{DL}} \mathbf{W}_2 & \mathbf{H}_2^{\mathrm{DL}} \mathbf{W}_3 \\ \mathbf{H}_3^{\mathrm{DL}} \mathbf{W}_1 & \mathbf{H}_3^{\mathrm{DL}} \mathbf{W}_2 & \mathbf{H}_3^{\mathrm{DL}} \mathbf{W}_3 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_1 \\ \tilde{\mathbf{x}}_2 \\ \tilde{\mathbf{x}}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix} \tag{13.17}$$

MIMO-OFDM Wireless Communications with MATLAB®
Yong Soo Cho, Jaekwon Kim, Won Young Yang, and Chung G. Kang

13.3.2 Block Diagonalization

$$\mathbf{H}_u^{\mathrm{DL}}\mathbf{W}_k = \mathbf{0}_{N_{M,u}\times N_{M,u}}, \forall u \neq k \tag{13.18}$$

$$\mathbf{y}_u = \mathbf{H}_u^{\mathrm{DL}}\mathbf{W}_u\tilde{\mathbf{x}}_u + \mathbf{z}_u, \quad u = 1, 2, \cdots, K \tag{13.19}$$

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}} = \left[\left(\mathbf{H}_1^{\mathrm{DL}}\right)^H \cdots \left(\mathbf{H}_{u-1}^{\mathrm{DL}}\right)^H \left(\mathbf{H}_{u+1}^{\mathrm{DL}}\right)^H \cdots \left(\mathbf{H}_K^{\mathrm{DL}}\right)^H\right]^H \tag{13.20}$$

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}}\mathbf{W}_u = \mathbf{0}_{(N_{M,total}-N_{M,u})\times N_{M,u}}, \quad u = 1, 2, \cdots, K \tag{13.21}$$

13.3.2 Block Diagonalization

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1^{\mathrm{DL}}\mathbf{W}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2^{\mathrm{DL}}\mathbf{W}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_3^{\mathrm{DL}}\mathbf{W}_3 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_1 \\ \tilde{\mathbf{x}}_2 \\ \tilde{\mathbf{x}}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix} \tag{13.22}$$

$$\tilde{\mathbf{H}}_u^{\mathrm{DL}} = \tilde{\mathbf{U}}_u \tilde{\Lambda}_u \left[\tilde{\mathbf{V}}_u^{\text{non-zero}} \; \tilde{\mathbf{V}}_u^{\text{zero}} \right]^H \tag{13.23}$$

$$\begin{aligned} \tilde{\mathbf{H}}_u^{\mathrm{DL}} \tilde{\mathbf{V}}_u^{\text{zero}} &= \tilde{\mathbf{U}}_u \left[\tilde{\Lambda}_u^{\text{non-zero}} \; \mathbf{0} \right] \begin{bmatrix} \left(\tilde{\mathbf{V}}_u^{\text{non-zero}} \right)^H \\ \left(\tilde{\mathbf{V}}_u^{\text{zero}} \right)^H \end{bmatrix} \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} \left(\tilde{\mathbf{V}}_u^{\text{non-zero}} \right)^H \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} \mathbf{0} \\ &= \mathbf{0} \end{aligned} \tag{13.24}$$

13.3.2 Block Diagonalization

$$\tilde{\mathbf{H}}_1^{\mathrm{DL}} = \tilde{\mathbf{U}}_1 \tilde{\Lambda}_1 \left[\tilde{\mathbf{V}}_1^{\text{non-zero}} \ \tilde{\mathbf{V}}_1^{\text{zero}} \right]^H$$
$$= [\tilde{\mathbf{u}}_{11} \quad \tilde{\mathbf{u}}_{12}] \begin{bmatrix} \tilde{\lambda}_{11} & 0 & 0 & 0 \\ 0 & \tilde{\lambda}_{12} & 0 & 0 \end{bmatrix} [\tilde{\mathbf{v}}_{11} \quad \tilde{\mathbf{v}}_{12} \quad \tilde{\mathbf{v}}_{13} \quad \tilde{\mathbf{v}}_{14}]^H \qquad (13.25)$$

$$\tilde{\mathbf{H}}_2^{\mathrm{DL}} = \tilde{\mathbf{U}}_2 \tilde{\Lambda}_2 \left[\tilde{\mathbf{V}}_2^{\text{non-zero}} \ \tilde{\mathbf{V}}_2^{\text{zero}} \right]^H$$
$$= [\tilde{\mathbf{u}}_{21} \quad \tilde{\mathbf{u}}_{22}] \begin{bmatrix} \tilde{\lambda}_{21} & 0 & 0 & 0 \\ 0 & \tilde{\lambda}_{22} & 0 & 0 \end{bmatrix} [\tilde{\mathbf{v}}_{21} \quad \tilde{\mathbf{v}}_{22} \quad \tilde{\mathbf{v}}_{23} \quad \tilde{\mathbf{v}}_{24}]^H \qquad (13.26)$$

13.3.2 Block Diagonalization

$$\begin{aligned}\mathbf{W}_1 &= \tilde{\mathbf{V}}_1^{\text{zero}} = [\tilde{\mathbf{v}}_{13} \quad \tilde{\mathbf{v}}_{14}] \\ \mathbf{W}_2 &= \tilde{\mathbf{V}}_2^{\text{zero}} = [\tilde{\mathbf{v}}_{23} \quad \tilde{\mathbf{v}}_{24}]\end{aligned} \tag{13.27}$$

$$\mathbf{x} = \mathbf{W}_1\tilde{\mathbf{x}}_1 + \mathbf{W}_2\tilde{\mathbf{x}}_2 \tag{13.28}$$

$$\begin{aligned}\mathbf{y}_1 &= \mathbf{H}_1^{\text{DL}}\mathbf{x} + \mathbf{z}_1 \\ &= \mathbf{H}_1^{\text{DL}}(\mathbf{W}_1\tilde{\mathbf{x}}_1 + \mathbf{W}_2\tilde{\mathbf{x}}_2) + \mathbf{z}_1 \\ &= \tilde{\mathbf{H}}_2^{\text{DL}}\left(\tilde{\mathbf{V}}_1^{\text{zero}}\tilde{\mathbf{x}}_1 + \tilde{\mathbf{V}}_2^{\text{zero}}\tilde{\mathbf{x}}_2\right) + \mathbf{z}_1 \\ &= \tilde{\mathbf{H}}_2^{\text{DL}}\tilde{\mathbf{V}}_1^{\text{zero}}\tilde{\mathbf{x}}_1 + \mathbf{z}_1 \\ &= \mathbf{H}_1^{\text{DL}}\tilde{\mathbf{V}}_1^{\text{zero}}\tilde{\mathbf{x}}_1 + \mathbf{z}_1\end{aligned} \tag{13.29}$$

13.3.2 Block Diagonalization

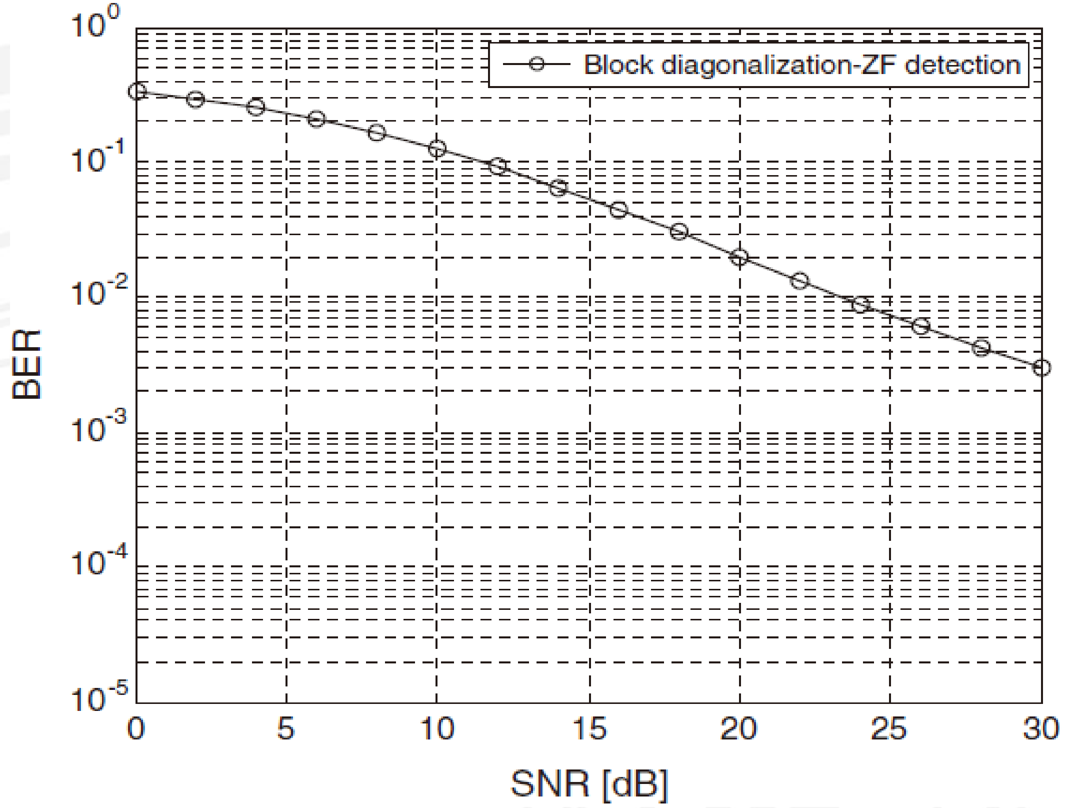

Figure 13.6 BER performance of block diagonalization method using zero-forcing detection at the receiver: $N_B = 4$, $K = 2$, and $N_{M,1} = N_{M,2} = 2$.

Kodlar

- Program 13.5 “Block_diagonalization.m” for BD method using zero-forcing detection